UNIT –I

INTRODUCTION TO ELECTRICAL ENGINEERING

Objectives:

- To introduce the terminology used in electrical circuits.
- To study voltage-current relationships of circuit elements.
- To analyze electrical circuits using techniques like voltage division, current division.

Syllabus: INTRODUCTION TO ELECTRICAL CIRCUITS

Introduction, History of Electrical Engineering, Network Elements classification, Circuit Concepts-R,L,C, Ideal Voltage Source, Practical Voltage Source, Ideal Current Source, Practical Current Source, Independent Voltage Source & Current Source, Dependent Voltage Source & Current Source, Voltage-Current Relationships for passive Elements.

Outcomes:

On completion, the student should be able to:

- Understand various terminology used in electrical circuits.
- Classify network elements and understand voltage-current relationships of R, L, C elements.
- Differentiate between ideal/practical/dependent and independent sources and their characteristics.

Learning Material

Introduction:

An Electric circuit is an interconnection of various elements in which there is at least one closed path in which current can flow. An Electric circuit is used as a component for any engineering system.

The performance of any electrical device or machine is always studied by drawing its electrical equivalent circuit. By simulating an electric circuit, any type of system can be studied for e.g., mechanical, hydraulic thermal, nuclear, traffic flow, weather prediction etc.

All control systems are studied by representing them in the form of electric circuits. The analysis, of any system can be learnt by mastering the techniques of circuit theory.

Electric circuit theory and electromagnetic theory are the two fundamental theories upon which all branches of electrical engineering are built. Many branches of electrical engineering, such as power, electric machines, control, electronics, communications, and instrumentation, are based on electric circuit theory. Therefore, the basic electric circuit theory course is the most important course for an electrical engineering student, and always an excellent starting point for a beginning student in electrical engineering education. Circuit theory is also valuable to students specializing in other branches of the physical sciences because circuits are a good model for the study of energy systems in general, and because of the applied mathematics, physics, and topology involved.

In electrical engineering, we are often interested in communicating or transferring energy from one point to another. To do this requires an interconnection of electrical devices. Such interconnection is referred to as an electric circuit, and each component of the circuit is known as an element.

Key Points

- The valance electrons which are loosely attached to the nucleus of an atom are called free electrons.
- The flow of free electrons is called as electric current.
- Time rate of change of charge is called as electric current.

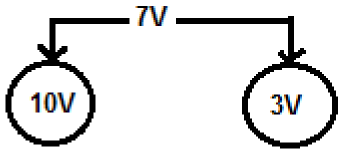
$$i = \frac{dQ}{dt} \text{Coulomb}/_{\text{sec}} \text{(or)Ampere}$$

If one coulomb charge flows through one section in one second is called as one Ampere current.

- Voltage is the energy required to move a unit charge through an element.

$$V = \frac{dW}{dQ} \text{Joule}/_{\text{Coulomb}} \text{(or)Volts}$$

- The difference in the potential of two charged bodies is called as potential difference.



Units: Volt

- Total work done in electric circuit is called as energy (E).

  Units: Joules

- Rate of transfer of energy is called as power (P).

$$P = \frac{dW}{dt}$$

$$P = \frac{dW}{dQ} * \frac{dQ}{dt}$$

$$P = V * I$$

"The rate at which work is done in electric circuit is called as power".

- **Electrical Network and Circuit**

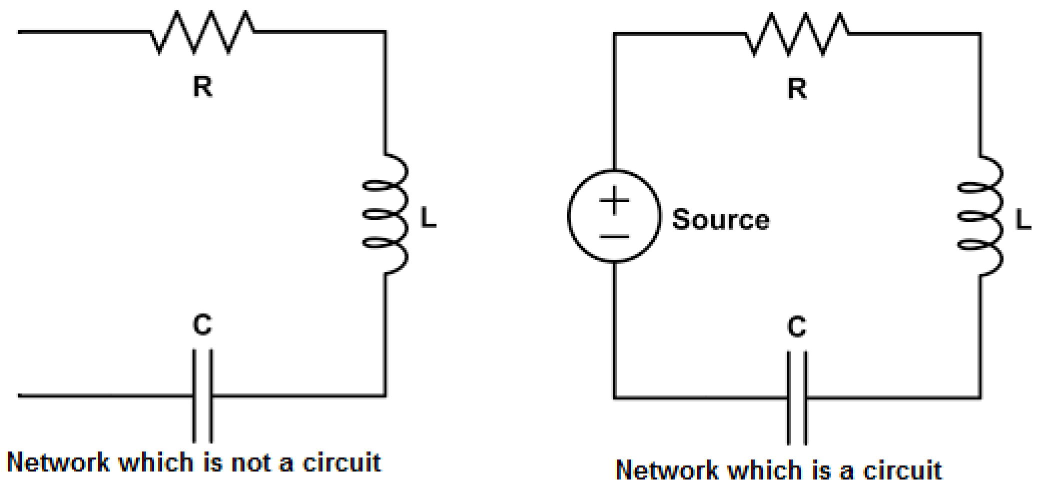


Fig 1.1 Shows Network and Circuit difference

The interconnection of two or more circuit elements (Sources, Resistors, inductors and capacitors) is called an Electric network. If the network contains at least one closed path, it is called an electric circuit. Every circuit is a network, but all networks are not circuits.

Introduction to Electrical Engineering :

Electrical engineering (sometimes referred to as electrical and electronic engineering) is a professional engineering discipline that deals with the study and application of electricity, electronics and electromagnetism. The field first became an identifiable occupation in the late nineteenth century with the commercialization of the electric telegraph and electrical power supply. The field now covers a range of sub-disciplines including those that deal with power, optoelectronics, digital electronics, analog electronics, computer science, artificial intelligence, control systems, electronics, signal processing and telecommunications.

The term electrical engineering may or may not encompass electronic engineering. Where a distinction is made, electrical engineering is considered to deal with the problems associated with large-scale electrical systems such as power transmission and motor control, whereas electronic engineering deals with the study of small-scale electronic systems including computers and integrated circuits. Another way of looking at the distinction is that electrical engineers are usually concerned with using electricity to transmit energy, while electronics engineers are concerned with using electricity to transmit information.

History of Electrical Engineering :

William Gilbert (1540–1603), English physician, founder of magnetic science, published De-Magnet, a treatise on magnetism, in 1600.

Charles A. Coulomb (1736–1806), French engineer and physicist, published the laws of electrostatics in seven memoirs to the French Academy of Science between 1785 and 1791. His name is associated with the unit of charge.

James Watt (1736–1819), English inventor, developed the steam engine. His name is used to represent the unit of power.

Alessandro Volta (1745–1827), Italian physicist, discovered the electric pile. The unit of electric potential and the alternate name of this quantity (voltage) are named after him.

Hans Christian Oersted (1777–1851), Danish physicist, discovered the connection between electricity and magnetism in 1820. The unit of magnetic field strength is named after him.

Andre Marie Ampere ` (1775–1836), French mathematician, chemist, and physicist, experimentally quantified the relationship between electric current and the magnetic field. His works were summarized in a treatise published in 1827. The unit of electric current is named after him.

Georg Simon Ohm (1789–1854), German mathematician, investigated the relationship between voltage and current and quantified the phenomenon of resistance. His first results were published in 1827. His name is used to represent the unit of resistance.

Michael Faraday (1791–1867), English experimenter, demonstrated electromagnetic induction in 1831. His electrical transformer and electromagnetic generator marked the beginning of the age of electric power. His name is associated with the unit of capacitance.

Joseph Henry (1797–1878), American physicist, discovered self-induction around 1831, and his name has been designated to represent the unit of inductance. He had also recognized the essential structure of the telegraph, which was later perfected by Samuel F. B. Morse.

Network Elements Classification :

1. **Active & passive:**

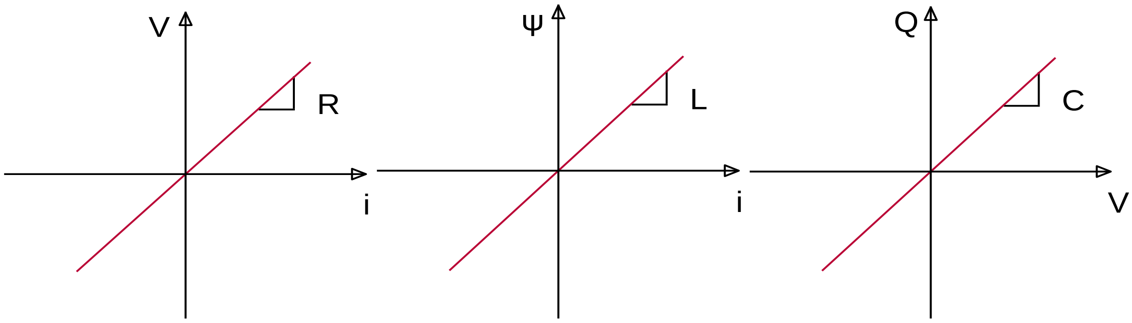
   An element is said to be active, if it is able to deliver the energy to outside world for infinite time, otherwise passive. Examples for active elements are sources and passive are R,L,C.

Note:

1. Ohm's Law is not applicable for active elements.
2. If V/I ratio is positive, then it is called as passive element.Passive elements cannot supply more energy than what it had drawn previously.

2. **Linear & Non-linear elements:**

   If the characteristic of an element is a straight line passing through the origin, it is called as linear element and these characteristics are constant.



Examples:

- Linear elements are R, L, and C.
- Non-Linear elements are Diode, Transistor.

3. Unilateral &Bilateral elements:

If an elements offers same impedance (opposition) for both the directions of flow of current through it is called as bilateral element otherwise it is unilateral element.

Examples:

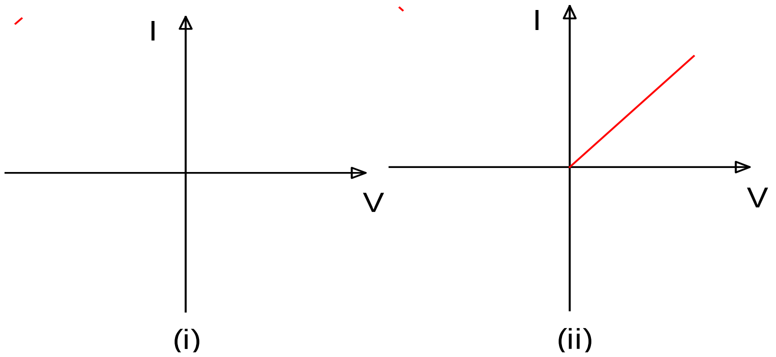
- Bilateral elements are R, L, C.
- Unilateral element is Diode, transistor.

For forward voltage Diode acts as short circuit. i.e. R=0. In reverse Bias it acts as open circuit i.e. R is infinity. So here Diode offers different resistance for different of current. Therefore, it is called as unilateral element.

If V/I characteristics are same in all direction, it is called as Bilateral element.

4. Time variant / invariant:

If the element characteristics are independent of time, it is called as time variant, otherwise time variant.



Case (i)

$\frac{V}{I}$ is Positive. ∴ It is passive element, bilateral element, linear element.

Case (ii)

$\frac{V}{I}$ is Positive in one Quadrant and $\frac{V}{I}$ is Negative in other direction.∴ $\frac{V}{I}$ ratio is not same in both directions.∴ It is active element, unilateral element, non-linear element.

5. **Lumped and Distributed Elements**

Lumped elements are those elements which are very small in size & in which simultaneous actions takes place. Typical lumped elements are capacitors, resistors, inductors.

Distributed elements are those which are not electrically separable for analytical purposes. For example a transmission line has distributed parameters along its length and may extend for hundreds of miles.

Circuit concepts :

Resistor :

Materials in general have a characteristic behavior of resisting the flow of electric charge. This physical property, or ability to resist current, is known as **resistance** and is represented by the symbol R.

The circuit element used to model the current- resisting behavior of a material is the **resistor**.

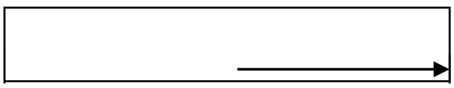
Resistance: (R)

It is a property of a material, which opposes the flow of electric current.

Units: ohm's Ω

Let ‘l’ be the length of the material

A be the cross sectional area of material.



Resistance is directly proportional to length of the material,

$$R \propto l \qquad (1.1)$$

As the area of cross section increases, electron can move freely.

∴Resistance is inversely proportional to the area of cross section.

$$R \propto \frac{1}{A} \qquad (1.2)$$

From (1) & (2)

$$R \propto \frac{l}{A}$$

$$R = \frac{\rho l}{A}$$

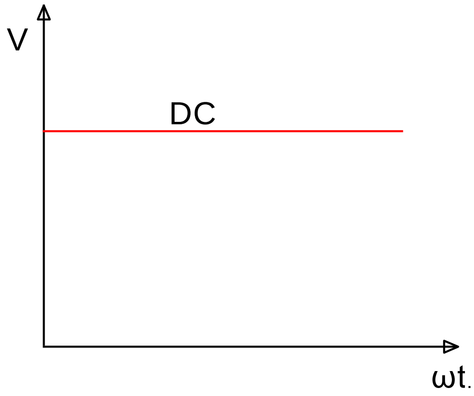
$$\rho = \text{Resistivity (or)Specific Resistance}$$

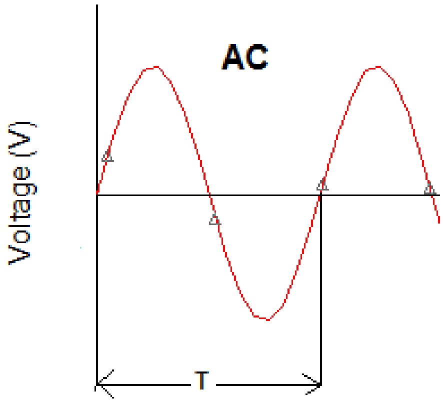
$$\rho = \frac{RA}{l} = \frac{\Omega * m^2}{m} = \Omega - m$$

Type of supplies:

Depends on the nature of the wave form power supplies are classified as

1. Alternating current (AC)
2. Direct current (DC)





DC is a current that remains constant with time.

AC is a current that varies sinusoidally with time.

The minimum time after which the cycle of signal repeats is called as time period (T).

	D.C	A.C
Representation	$V = K$	$V = ASin\omega t$ Where $\omega = 2\pi f$
Time period	∞	T
Frequency	0	$\frac{1}{T}$

Faraday's Laws:

First law:

Whenever conductor experiences the rate of change of flux, emf will be induced in that conductor and if there is a closed path, current will flow in that circuit.

Second Law:

The induced emf (e) is proportional to rate of change of flux.

$$e \propto \frac{d\emptyset}{dt} \rightarrow \text{for one turn}$$

If N turns are there, then

$$e \propto N\frac{d\emptyset}{dt}$$

$$e = -N\frac{d\emptyset}{dt}$$

$$e = -\frac{d(N\emptyset)}{dt}$$

$$e = -\frac{d(\psi)}{dt}$$

Here ψ is flux linkage, where $\psi = N\emptyset$

Here –ve sign indicates that induced emf opposes the current in that conductor which is given by Lenz's Law.

Lenz's Law:

The effect opposes the cause.

Inductors:

"An inductor consists of a coil of conducting wire".

Any conductor of electric current has inductive properties and maybe regarded as an inductor. But in order to enhance the inductive effect,a practical inductor is usually formed into a cylindrical coil with manyturns of conducting wire.

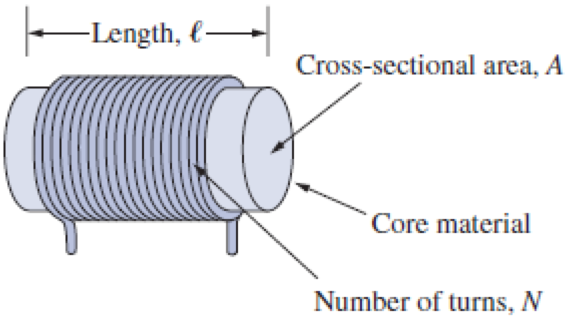


Fig.1.2 Typical form of an Inductor

Applications:

Inductors find numerous applications in electronic and power systems. They are used in power supplies, transformers, radios, TVs, radars, and electric motors.

Inductance:

"The property of coil that opposes any change in the amount of current flowing through it is called as Inductance".

Flux linkage depends on the amount of current flowing through the coil.

$$\therefore \ \psi \propto i$$

$$\psi = Li \ [L\text{=Inductance of coil}]$$

According to Faraday's Law

$$e = \frac{d(\psi)}{dt} = \frac{d}{dt}(Li)$$

$e = L\frac{di}{dt}$

$$\boxed{e = L\frac{di}{dt}}$$

According to Lenz's Law, induced emf should oppose the change in current flow through that coil.

The direction of induced voltage is given by,

Energy stored in the inductance $(E) = \int P\,dt$

$$= \int P\,dt$$

$$= \int vi\,dt$$

$$= \int L\frac{di}{dt}i\,dt$$

$$= \frac{L}{2}\int (2i)\frac{di}{dt}\,dt$$

$$= \frac{L}{2} \int (2i) di$$

$$E = \frac{1}{2} Li^2$$

$$\boxed{E = \frac{1}{2} Li^2}$$

Properties of inductor:

1. $V = L \frac{di}{dt}$

   The induced voltage across an inductor is zero if the current through it is constant. That means an inductor acts as **short circuit** to **dc**.

2. For small change in current within zero time (dt = 0) gives an infinite voltage across the inductor which is physically not at all feasible.

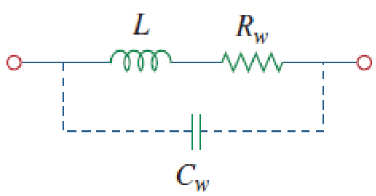
3. In an inductor, the current cannot change abruptly. Since it does not allow the sudden change in current through it, it is called as current stiff element.

4. An inductor behaves as **open circuit just** after **switching** across **dc voltage**.

5. The inductor can store finite amount of energy, even if the voltage across the inductor is zero. It stores the energy in the form of **magnetic field**.

6. The ideal inductor does not dissipate energy. The energy stored in it can be retrieved at a later time. The inductor takes power from the circuit when storing energy and delivers power to the circuit when returning previously stored energy. However, physical inductor dissipates power due to internal resistance.

7. A practical, non-ideal inductor has a significant resistive component, as shown in Figure below. This is due to the fact that the inductor is made of a conducting material such as copper, which has some resistance. This resistance is called the winding resistanceR_w, and it appears in series with the inductance of the inductor. The presenceR_w of makes it both an energy storage device and an energy dissipation device. SinceR_w is usually very small, it is ignored in most cases. The non-ideal inductor also has a winding capacitance$C_w$due to the capacitive coupling between the conducting coils. $C_w$is very small and can be ignored in most cases, except at high frequencies. So, we will assume ideal inductors.



Capacitor:

Any two conducting surfaces separated by an insulating material (dielectric) is called as capacitor.

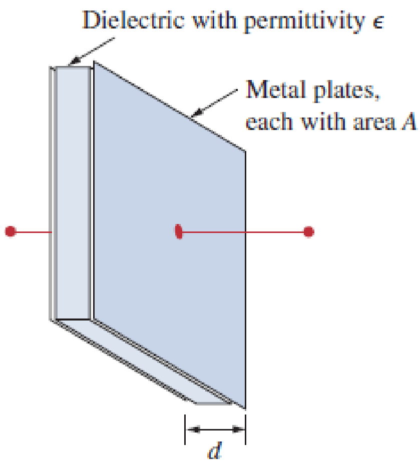


Fig.1.3.a A typical Capacitor

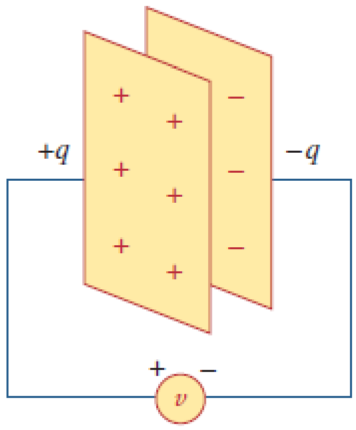


Fig.1.3.b A capacitor with applied voltage *v*.

Applications:

Capacitors are used extensively in electronics, communications, computers, and power systems. For example, they are used in the tuning circuits of radio receivers and as dynamic memory elements in computer systems.

Capacitance:

The ability of a capacitor to store charge is known as its capacitance.

Charge stored in capacitor is proportional to applied voltage.

$$\therefore Q \propto V$$

$$Q = CV$$

$$C = \frac{Q}{V}$$

$$\text{We know that, } i = \frac{dQ}{dt} = \frac{d}{dt}(CV)$$

$$i = C\frac{dV}{dt}$$

Although the capacitance C of a capacitor is the ratio of the charge Q_{per} plate to the applied voltage V it does not depend on Q or V. It depends on the physical dimensions of the capacitor. For example, for the parallel-plate capacitor shown in Fig., the capacitance is given by

$$C = \frac{\epsilon A}{d}$$

Where, A = surface area of each plate,

d = the distance between the plates,

ε = the permittivity of the dielectric material between the plates.

Energy stored in capacitor:

Let us consider 'V' voltage is applied across capacitor. At this instant, 'W' joules of work will be done in transferring 1C of charge from one plate to other.

If small charge dq is transferred, then work done is

$$dW = Vdq$$

$$W = \int_0^V CVdq$$

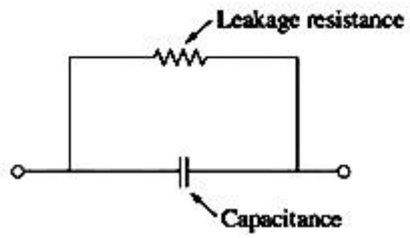
$$W = \frac{1}{2}CV^2$$

$$W = \frac{1}{2}C\left[\frac{q}{C}\right]^2$$

$$W = \frac{1}{2}\frac{q^2}{C}$$

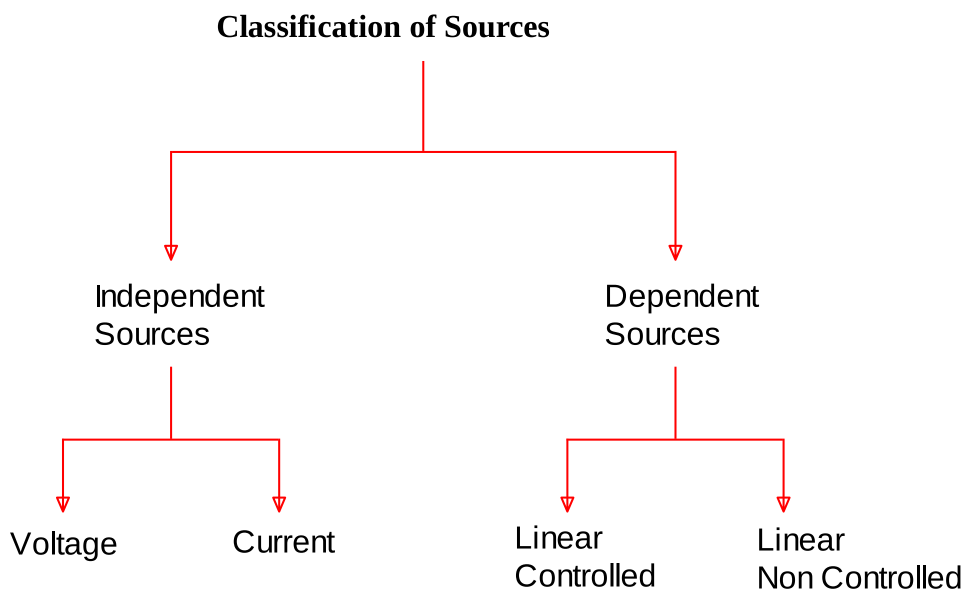
Properties of capacitor:

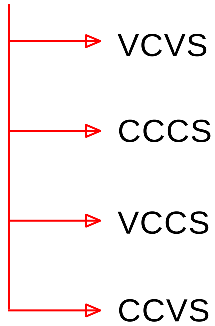
1. It stores energy in the form of electrostatic field.
2. The current in a capacitor is zero, if the voltage across it is constant, that means the capacitor acts as an **open circuit** to **dc**.
3. A small change in voltage across a capacitance within zero time gives an infinite current through the capacitor, which is physically impossible.
   i. Capacitor doesn't allow the sudden change in voltage. So, it is called as voltage stiff element.
   ii. A capacitor behaves as **short circuit just** after **switching** across **dc** voltage.
4. The capacitor can store a finite amount of energy, even if the current through it is zero.
5. A pure (or Ideal) capacitor never dissipates energy but only stores it hence it is called non-dissipative element.
6. A real, nonideal capacitor has a parallel-model leakage resistance, as shown in Fig. below. The leakage resistance may be as high as 100 MΩ and can be neglected for most practical applications consider as an Ideal Capacitor.



Circuit model of a nonideal capacitor.

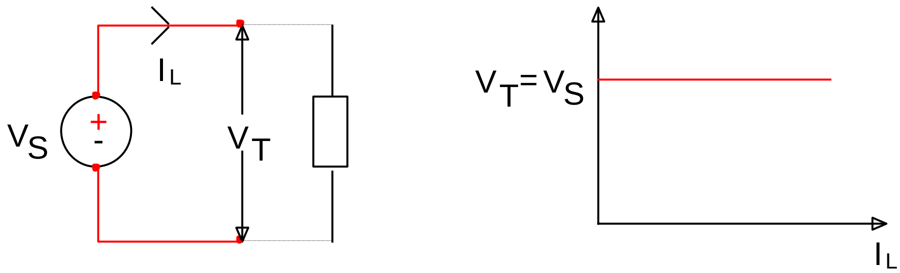
Voltage and Current Sources :



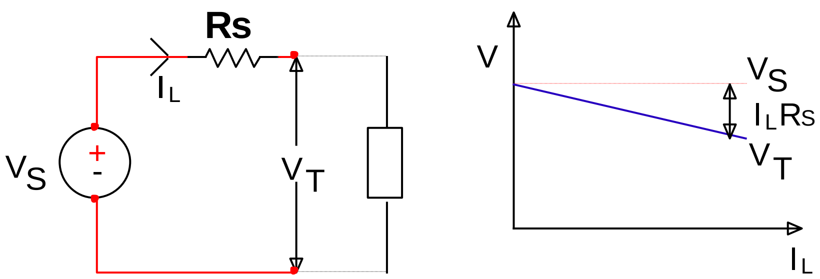


Independent Voltage Source

When the terminal voltage of a source is independent of load element, it is called as independent ideal voltage source. (Or) Ideal voltage source is one which delivers energy to the load at a constant terminal voltage, irrespective of the current drawn by the load.



Practical voltage source having, its internal resistance(R_s). Whenever load current increases, the drop across R_s will increase. Therefore, terminal voltage will reduce as load current rises.

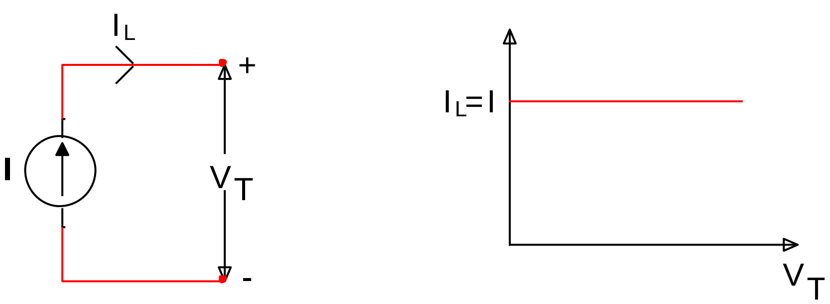


$$V_T = V_S - I_S R_S$$

Ideal Current Source.

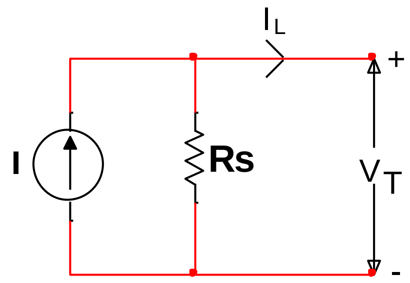
It is a two terminal device which delivers constant current to the network connected across its terminals. i.e. current supplied by the source is independent of its terminal voltage. (Or) An ideal current source is one, which delivers energy with a constant current to the load, irrespective of the terminal voltage across the load.

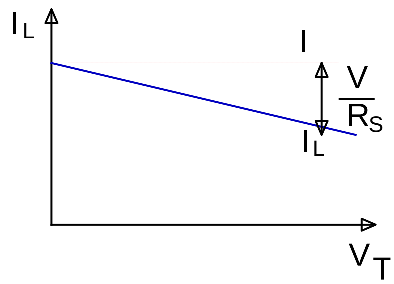
Practical Current Source



A Practical source always possesses a very small value of internal resistance r. The internal resistance of a voltage source is always connected in series with it & for a current source; it is always connected in parallel with it.

As the value of the internal resistance of a practical current source is very small, a practical current source is also assumed to deliver a constant current, irrespective of the terminal voltage across the load connected to it.

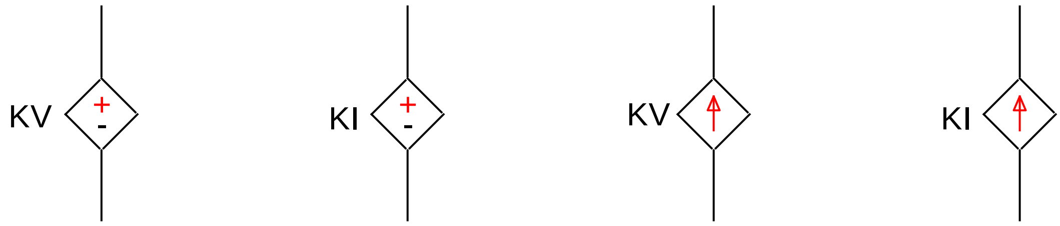




Dependent (or) Controlled Source

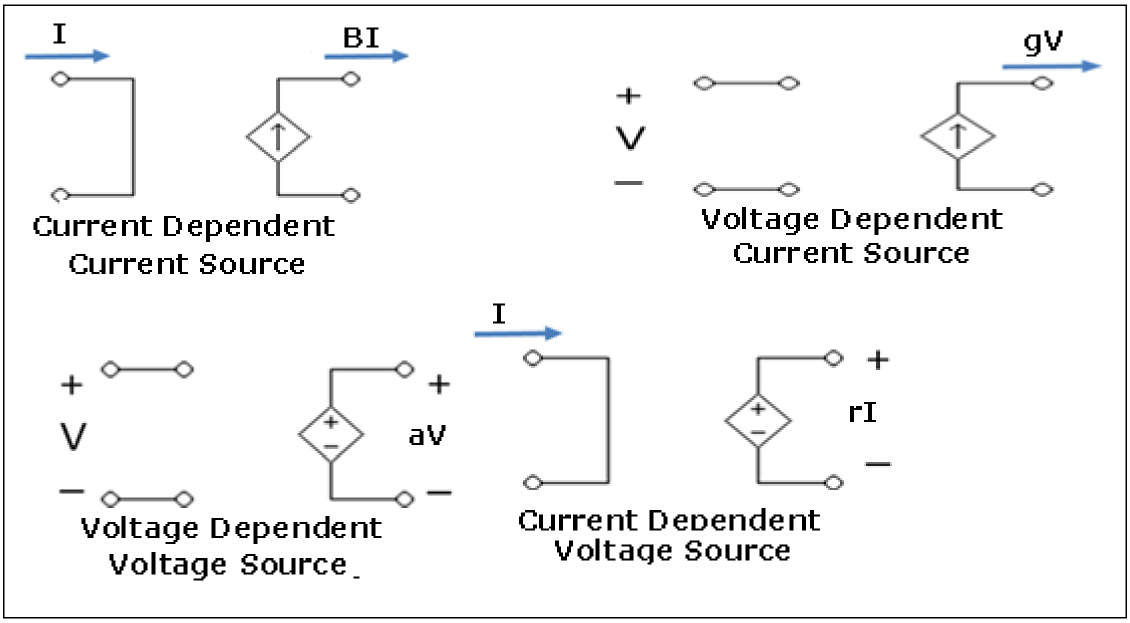
A controlled voltage/ current source is one whose terminal voltage or current is a function of some other voltage or current. These devices have two pairs of terminals. One pair corresponds to the controlling quantity & other pair represents controlled quantity.

Controlled quantity is directly proportional to controlling quantity.



Voltage controlled Voltage source | Current controlled Voltage source | Voltage controlled Current source | Current controlled Current source

Here, $K = \text{Constant}$



The constants of proportionalities are written as B, g, a, r in which B & a has no units, r has units of ohm & g units of mhos.

Independent sources actually exist as physical entities such as battery, a dc generator & an alternator. But dependent sources are used to represent electrical properties of electronic devices such as OP-AMPS & Transistors.

V-I Relation for passive elements

Circuit elements	Voltage(V)	Current(A)	Power(W)
Resistor R (Ohms Ω)	$V = RI$	$I = \frac{V}{R}$	$P = i^2R$
Inductor L (Henry H)	$V = L\frac{di}{dt}$	$I = \frac{1}{L}\int vdt + i_0$ Where i_0 is the initial currentthrough inductor	$P = Li\frac{di}{dt}$
Capacitor C (Farad F)	$I = \frac{1}{C}\int idt + v_0$ Where v_0 is the initial voltage across capacitor	$I = C\frac{dv}{dt}$	$P = CV\frac{dv}{dt}$

Elements of Electrical Circuits

UNIT – II

Network Equations and Reduction Techniques

Objectives:

- To analyze electrical circuits using voltage division
- To analyze electrical circuits using current division
- To analyze electrical circuits using reduction techniques techniques.
- To analyze electrical circuits using Mesh and Nodal methods.

Syllabus: Network Equations and Reduction Techniques

Ohms Law, Kirchhoff's Voltage Law, Kirchhoff's Current Law, Source Transformation, - network reduction Techniques Series, Parallel, and Series Parallel, Star to Delta & Delta to Star Transformations, Nodal Analysis, Mesh Analysis, Super node, Super mesh for Dc Excitations.

Outcomes:

On completion, the student should be able to:

- Understand various terminology used in electrical circuits.
- Classify network elements and understand voltage-current relationships of R, L, C elements.
- Understand Kirchhoff's Laws and Solve problems using voltage and current division techniques.
- Differentiate between ideal/practical/dependent and independent sources and their characteristics.
- Solve problems using source transformation technique, Mesh & Nodal methods of analysis.

Learning Material

Ohm's Law:

"Under constant temperature and pressure, current flowing through a conductor is directly proportional to the voltage applied across it".

$$i \propto V$$

$$i = \frac{V}{R}$$

Where, R=Resistance of conductor

$$\text{Power dissipated by Resistor (P)} = V * i$$

$$= \frac{V^2}{R} (or) i^2R$$

$$\text{If V is positive, then } P = \frac{(+V)^2}{R} = \frac{V^2}{R}$$

$$= (+i)^2R = i^2R \qquad (1.3)$$

$$\text{If V is Negative, then } P = (-i)^2R = i^2R$$

$$= \frac{(-V)^2}{R} = \frac{V^2}{R} \qquad (1.4)$$

Conclusion:

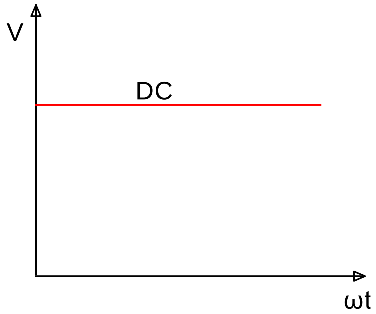
From (1.3) & (1.4) power dissipated by resistor remains same.

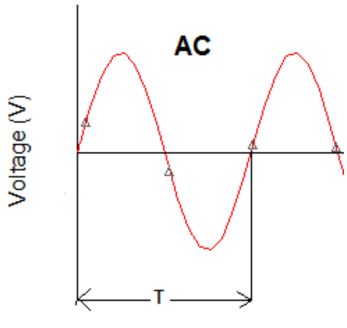
It is independent of direction of applied voltage (or) current.

Type of supplies:

Depends on the nature of the wave form power supplies are classified as

1. Alternating current (AC)
2. Direct current (DC)





DC is a current that remains constant with time.

AC is a current that varies sinusoidally with time.

The minimum time after which the cycle of signal repeats is called as time period (T).

	D.C	A.C
Representation	$V = K$	$V = ASin\omega t$ Where $\omega = 2\pi f$
Time period	∞	T
Frequency	0	$\frac{1}{T}$

Kirchhoff's Laws:

For analyzing a large variety of electric circuits. These laws are formally known as Kirchhoff's current law (KCL) and Kirchhoff's voltage law (KVL).

Kirchhoff's voltage Law: (KVL)

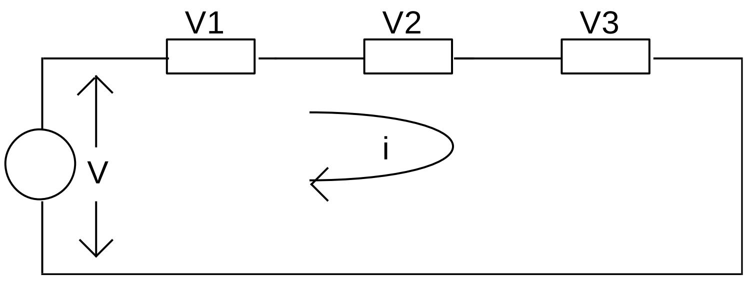
This law is related to emf's and voltage drops in a circuit. It stated as"in an electrical circuit, algebraic sum of all the voltages in a closed path is Zero".

$$-V + V_1 + V_2 + V_3 = 0$$

Or, Sum of voltage drops=Sum of voltage rises

$$V = V_1 + V_2 + V_3$$

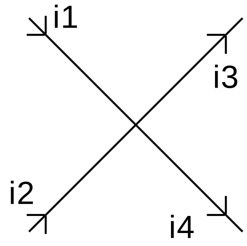
$$\boxed{V = V_1 + V_2 + V_3}$$



- KVL is independent of nature of element.

Kirchhoff's current Law:

This law is related to current at the junction points a circuit. It is stated as "In a circuit, at node at any instant the algebraic sum of current flowing towards a junction in circuit is Zero".



$$i_1 + i_2 - i_3 - i_4 = 0$$

$$\frac{dQ_1}{dt} + \frac{dQ_2}{dt} - \frac{dQ_3}{dt} - \frac{dQ_4}{dt} = 0$$

$$Q_1 + Q_2 - Q_3 - Q_4 = 0$$

- According to law of conservation of energy, the net charge at node is Zero.

- KCL is independent of nature of element.

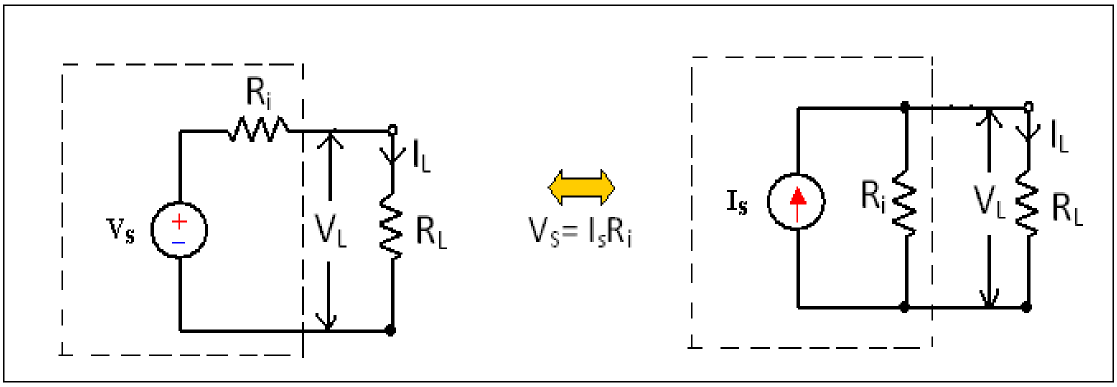
An alternative form of KCL, "The sum of the currents entering a node is equal to the sum of the currents leaving the node".

$$i_1 + i_2 = i_3 + i_4$$

Source transformation

A current source or a voltage source drives current through its load resistance and the magnitude of the current depends on the value of the load resistance.

Consider a practical voltage source and a practical current source connected to the same load resistance R_L as shown in the figure



R_i in figure represents the internal resistance of the voltage source V_S and current source I_S. Two sources are said to be identical, when they produce identical terminal voltage V_L and load current I_L.

The circuits in figure represent a practical voltage source & a practical current source respectively, with load connected to both the sources. The terminal voltage V_L and load currentI_L across their terminals are same.

Hence the practical voltage source & practical current source shown in the dotted box of figure are equal. The two equivalent sources should also provide the same open circuit voltage & short circuit current.

From fig (a)

$$I_L = \frac{V_S}{(R_i+R_L)}$$

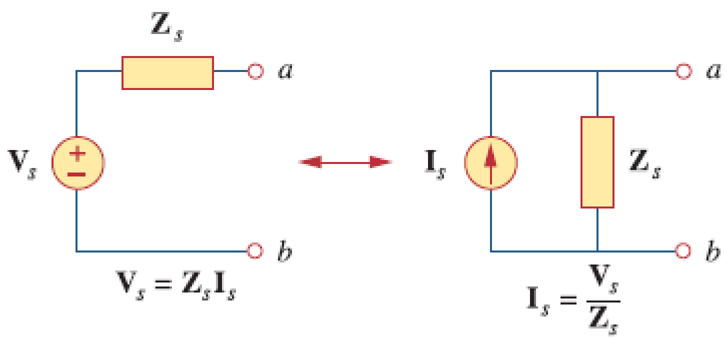
From fig (b)

$$I_L = I\frac{R_i}{R_i+R_L}$$

$$\because \frac{V_S}{(R_i + R_L)} = I\frac{R_i}{(R_i + R_L)}$$

$$V_s = IR_i \ \text{ or } \ I = \frac{V_S}{R_i}$$

Hence a voltage source V_S in series with its internal resistance R_i can be converted into a current source $I = \frac{V_S}{R_i}$, with its internal resistance R_i connected in parallel with it. Similarly a current source I in parallel with its internal resistance R_i can be converted into a voltage source $V = IR_i$ in series with its internal resistance R_i.



Network Reduction Techniques

Series and Parallel connection

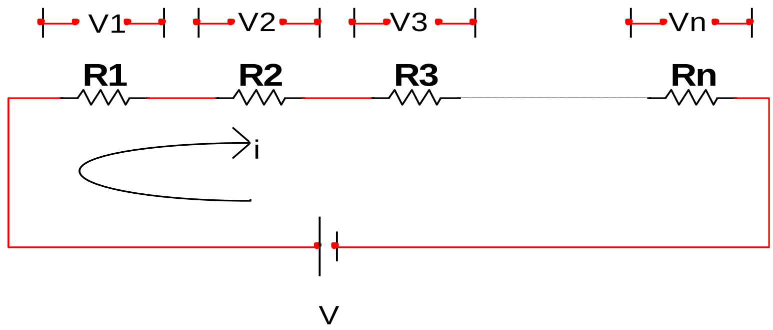
Two or more elements are in series if they exclusively share a single node and consequently carry the same current. Two or more elements are in parallel if they are connected to the same two nodes and consequently have the same voltage across them.

Elements are in series when they are chain-connected or connected sequentially, end to end. For example, two elements are in series if they share one common node and no other element is connected to that common node. Elements in parallel are connected to the same pair of terminals.

Series Resistors circuit:

Let us consider ‘n’ Resistors are connected in series.

Apply KVL



$$-V + V_1 + V_2 + V_3 + \cdots\cdots\cdots + V_n = 0$$

$$-iR_{eq} + iR_1 + iR_2 + iR_3 + \cdots\cdots\cdots + iR_n = 0$$

$$R_{eq} = R_1 + R_2 + R_3 + \cdots\cdots\cdots + R_n$$

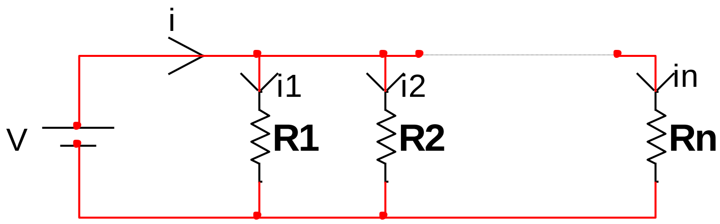
Note: If ‘n’ Resistors are in series, then equivalent Resistance will be greaterthan$R_1, R_2, R_3 \ldots R_n$.

Parallel Resistors circuit:

Apply KCL

$$-i + i_1 + i_2 + i_3 + \cdots\cdots + i_n = 0$$

$$-\frac{V}{R_{eq}} + \frac{V}{R_1} + \frac{V}{R_2} + \cdots\cdots + \frac{V}{R_n} = 0$$



- When ‘n’ Resistances are in parallel, equivalent Resistance is smaller than all Resistances.

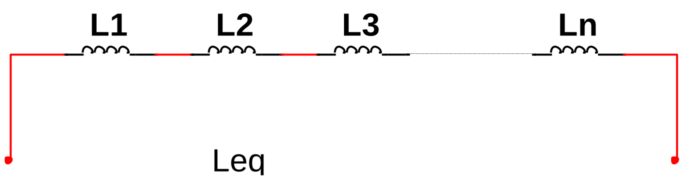
NOTE:

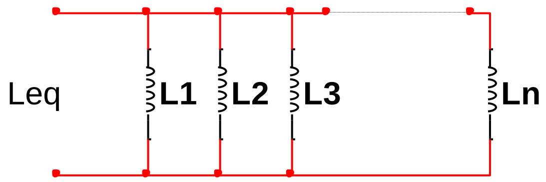
- When ‘n’ Resistances are in series, the current through all the Resistors are same.
- When ‘n’ Resistors are in parallel, then voltage across all resistors is same.

Inductive circuits:

Series Inductors circuit:

$$L_{eq} = L_1 + L_2 + \cdots\cdots + L_n$$





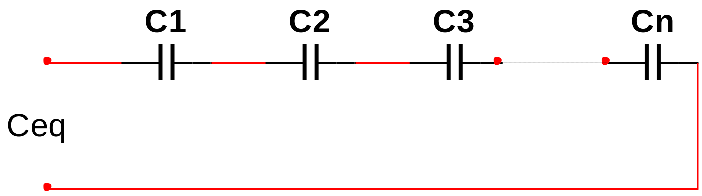
Parallel Inductors circuit:

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \cdots\cdots + \frac{1}{L_n}$$

Capacitive circuits:

Series circuit:

Apply KVL



$$V = V_1 + V_2 + V_3 + \cdots\cdots + V_n$$

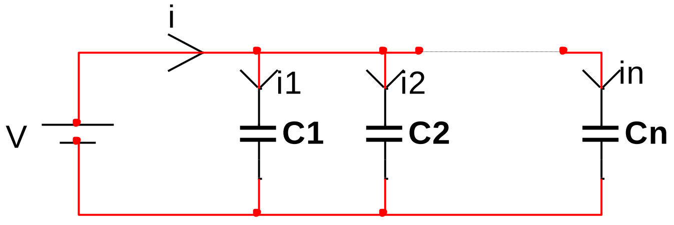
$$\frac{Q}{C_{eq}} = \frac{Q}{C_1} + \frac{Q}{C_2} + \cdots\cdots + \frac{Q}{C_n}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots\cdots + \frac{1}{C_n}$$

$$\boxed{\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots\cdots + \frac{1}{C_n}}$$

Parallel circuit:

Apply KCL



$$-i + i_1 + i_2 + i_3 + \cdots\cdots + i_n = 0$$

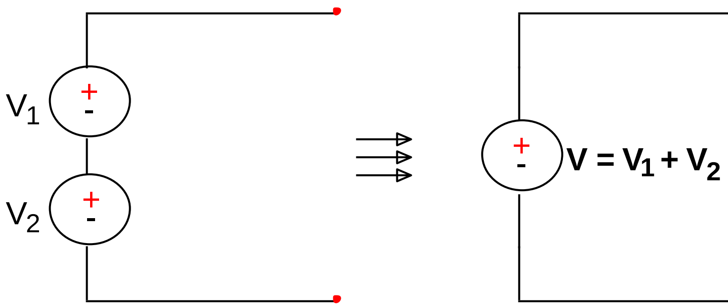
$$-Q + Q_1 + Q_2 + \cdots\cdots + Q_n = 0$$

$$-C_{eq}V + C_1V + C_2V + \cdots\cdots + C_nV = 0$$

$$-C_{eq} + C_1 + C_2 + \cdots\cdots + C_n = 0$$

$$\boxed{C_{eq} = C_1 + C_2 + \cdots\cdots + C_n}$$

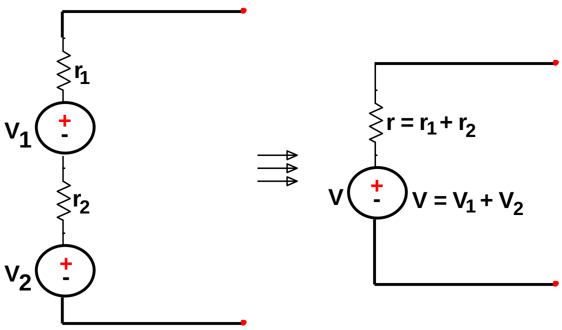
Ideal voltage source connected in series



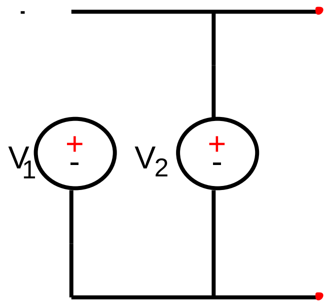
The equivalent single ideal voltage source is given by $V = V_1 + V_2$

Any number of ideal voltage sources connected in series can be represented by a single ideal voltage sum taking in to account the polarities connected together in to consideration.

Practical voltage source connected in series



Ideal voltage source connected in parallel

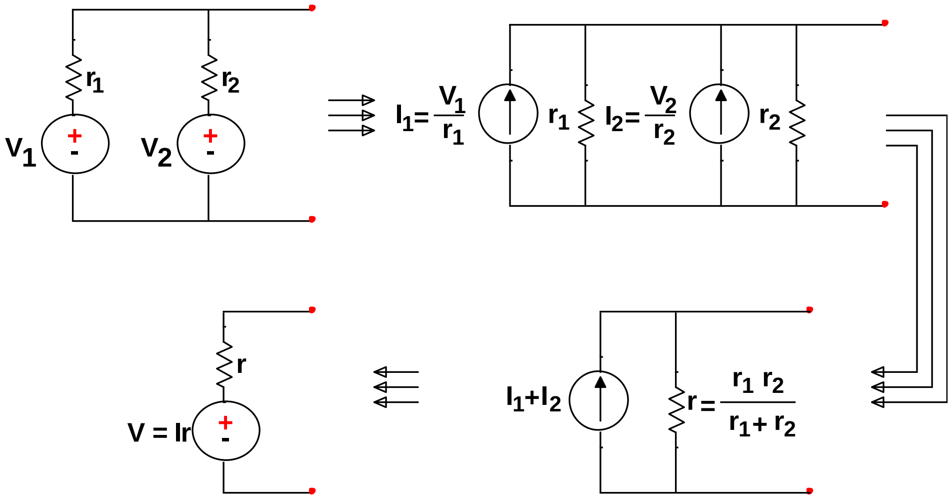


When two ideal voltage sources of emf's V_1 & V_2 are connected in parallel, what voltage appears across its terminals is ambiguous. Hence such connections should not be made.

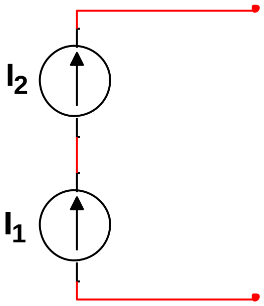
However if $V_1 = V_2 = V$, then the equivalent voltage some is represented by V.

In that case also, such a connection is unnecessary as only one voltage source serves the purpose.

Practical voltage sources connected in parallel



Ideal current sources connected in series

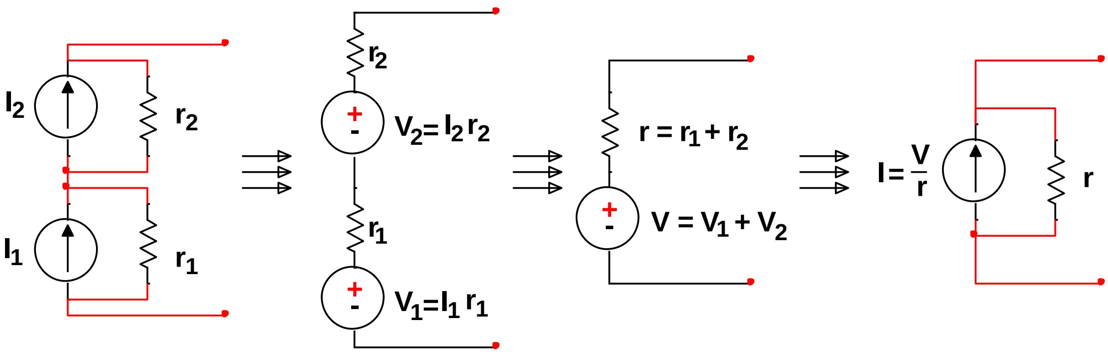


When ideal current sources are connected in series, what current flows through the line is ambiguous. Hence such a connection is not permissible.

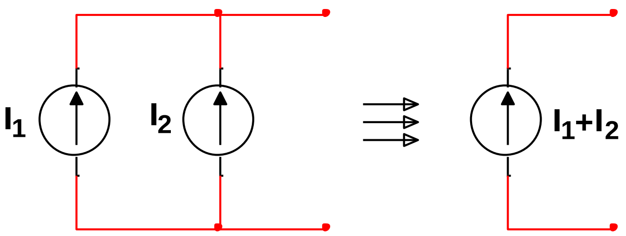
However, if $I_1 + I_2 = I$, then the current in the line is I.

But, such a connection is not necessary as only one current source serves the purpose.

Practical current sources connected in series:

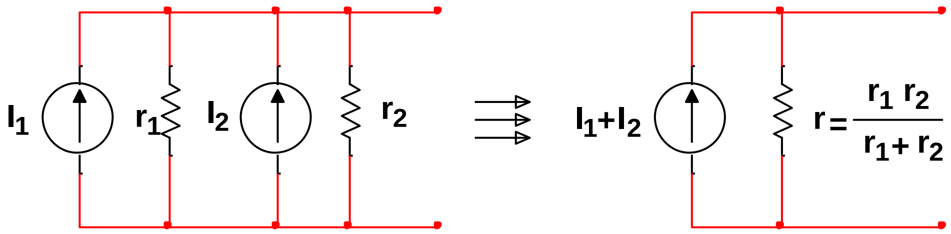


Ideal current sources connected in parallel



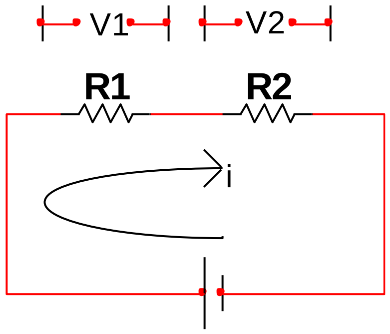
Two ideal current sources in parallel can be replaced by a single equivalent ideal current source.

Practical current sources connected in parallel



Voltage division Rule:

It is applicable for series circuit.



$$i = \frac{V}{R_1 + R_2}$$

$$V_1 = iR_1 = \left(\frac{V}{R_1 + R_2}\right) R_1$$

$$V_2 = iR_2 = \left(\frac{V}{R_1 + R_2}\right) R_2$$

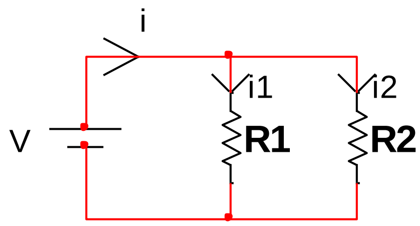
i.e When 'n' Resistors R_1, R_2, R_3… R_n are in series and V $V_1, V_2, V_3, \cdots\cdots V_n$ are voltage drops across resistors, then

$$V_1 = \left(\frac{V}{R_1 + R_2 + \cdots\cdots + R_n}\right) R_1$$

$$V_n = \left(\frac{V}{R_1 + R_2 + \cdots\cdots + R_n}\right) R_n$$

$$\boxed{V_n = \left(\frac{V}{R_1 + R_2 + \cdots\cdots + R_n}\right) R_n}$$

Current division Rule:



$$i = i_1 + i_2 + i_3 + \cdots\cdots + i_n$$

$$R_{eq} = \frac{V}{i} = \frac{R_1 R_2}{R_1 + R_2}$$

$$i = \frac{V}{R_{eq}} = \frac{V(R_1 + R_2)}{R_1 R_2}$$

$$i_2 = \frac{V}{R_2} = \frac{V(R_1 + R_2)R_1}{R_1 R_2 (R_1 + R_2)}$$

$$i_2 = \frac{V(R_1 + R_2)}{R_1 R_2} * \frac{R_1}{R_1 + R_2}$$

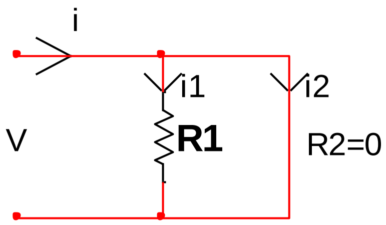
$$i_2 = \frac{i * R_1}{R_1 + R_2}$$

$$i_1 = \frac{i * R_2}{R_1 + R_2}$$

$$\boxed{i_2 = \frac{i * R_1}{R_1 + R_2} \& i_1 = \frac{i * R_2}{R_1 + R_2}}$$

Case (i):

$$i_1 = \frac{i * R_2}{R_1 + R_2} = 0$$



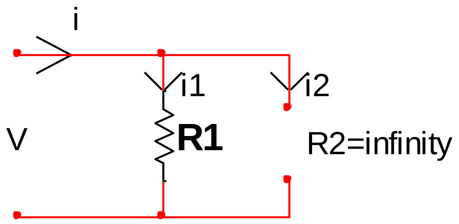
$$i_2 = \frac{i * R_1}{R_1 + R_2} = i$$

Observation: Current always choose lower Resistance path.

Case (ii)

$$i_1 = \frac{i * R_2}{R_1 + R_2} = i$$

$$i_2 = \frac{i * R_1}{R_1 + R_2} = 0$$

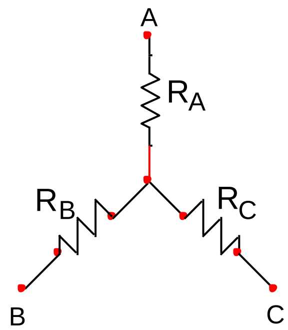


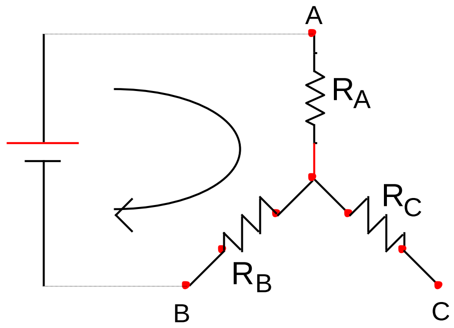
Note: Current will not flow through open circuit.

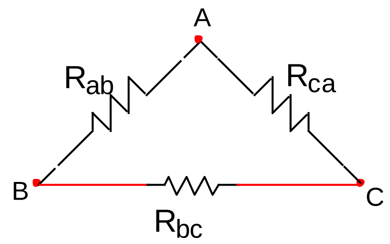
Star – Delta / Delta – Star Transformation ($\curlywedge - \Delta$ / $\Delta - \curlywedge$)

Let us consider three resistors are connected in star between the points A, B, C. So these resistors considered as R_A, R_B, R_C. R_{AB}, R_{BC}, R_{CA} be the resistances in Delta.

From the star connection,





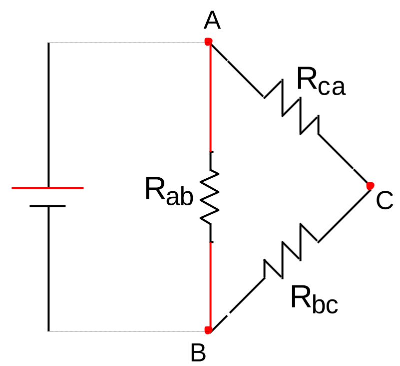


$$R_{AB} = R_A + R_B \qquad (1.6)$$

In the same way

$$R_{BC} = R_B + R_C \qquad (1.7)$$

$$R_{CA} = R_C + R_A \qquad (1.8)$$



From Delta connection

$$R_{AB} = R_{ab} \parallel (R_{ac} + R_{bc})$$

$$R_{AB} = \frac{R_{ab} * (R_{ac} + R_{bc})}{R_{ab} + R_{bc} + R_{ca}} \qquad (1.9)$$

In the same way

$$R_{BC} = R_{bc} \parallel (R_{ab} + R_{ac})$$

$$R_{BC} = \frac{R_{bc} * (R_{ab} + R_{ac})}{R_{ab} + R_{bc} + R_{ca}} \qquad (1.10)$$

$$R_{CA} = R_{ca} \parallel (R_{ab} + R_{bc})$$

$$R_{CA} = \frac{R_{ca}*(R_{ab}+R_{bc})}{R_{ab}+R_{bc}+R_{ca}} \quad (1.11)$$

From (1.6) & (1.9)

$$R_A + R_B = \frac{R_{ab}*(R_{ac}+R_{bc})}{R_{ab}+R_{bc}+R_{ca}} \quad (1.12)$$

From (1.7) & (1.10)

$$R_B + R_C = \frac{R_{bc}*(R_{ab}+R_{ac})}{R_{ab}+R_{bc}+R_{ca}} \quad (1.13)$$

From (1.8) & (1.11)

$$R_C + R_A = \frac{R_{ca}*(R_{ab}+R_{bc})}{R_{ab}+R_{bc}+R_{ca}} \quad (1.14)$$

(1.12) - (1.13) + (1.14), then

$$2R_A = \frac{R_{ab}R_{ac} + R_{ab}R_{bc} - R_{bc}R_{ab} - R_{bc}R_{ac} + R_{ca}R_{ab} + R_{ca}R_{bc}}{R_{ab} + R_{bc} + R_{ca}}$$

$$2R_A = \frac{2R_{ab}R_{ac}}{R_{ab} + R_{bc} + R_{ca}}$$

$$R_A = \frac{R_{ab}R_{ac}}{R_{ab}+R_{bc}+R_{ca}} \quad (1.15)$$

$$\boxed{R_A = \frac{R_{ab}R_{ac}}{R_{ab} + R_{bc} + R_{ca}}}$$

Substitute (1.15) in (1.12) & (1.14)

$$R_A + R_B = \frac{R_{ab} * (R_{ac} + R_{bc})}{R_{ab} + R_{bc} + R_{ca}}$$

$$\frac{R_{ab}R_{ac}}{R_{ab} + R_{bc} + R_{ca}} + R_B = \frac{R_{ab} * (R_{ac} + R_{bc})}{R_{ab} + R_{bc} + R_{ca}}$$

$$R_B = \frac{R_{ab} * (R_{ac} + R_{bc})}{R_{ab} + R_{bc} + R_{ca}} - \frac{R_{ab}R_{ac}}{R_{ab} + R_{bc} + R_{ca}}$$

$$R_B = \frac{R_{ab}R_{bc}}{R_{ab}+R_{bc}+R_{ca}} \quad (1.16)$$

$$\boxed{R_B = \frac{R_{ab}R_{bc}}{R_{ab} + R_{bc} + R_{ca}}}$$

$$R_C + R_A = \frac{R_{ca} * (R_{ab} + R_{bc})}{R_{ab} + R_{bc} + R_{ca}}$$

$$R_C + \frac{R_{ab}R_{ac}}{R_{ab} + R_{bc} + R_{ca}} = \frac{R_{ca} * (R_{ab} + R_{bc})}{R_{ab} + R_{bc} + R_{ca}}$$

$$R_C = \frac{R_{ca} * (R_{ab} + R_{bc})}{R_{ab} + R_{bc} + R_{ca}} - \frac{R_{ab}R_{ac}}{R_{ab} + R_{bc} + R_{ca}}$$

$$R_C = \frac{R_{ca}R_{bc}}{R_{ab}+R_{bc}+R_{ca}} \quad (1.17)$$

$$\boxed{R_C = \frac{R_{ca}R_{bc}}{R_{ab} + R_{bc} + R_{ca}}}$$

$$\frac{(1.15)*(1.16)}{(1.17)} \Longrightarrow \frac{R_A R_B}{R_C} = \frac{\frac{R_{ab}R_{ac}}{R_{ab}+R_{bc}+R_{ca}} * \frac{R_{ab}R_{bc}}{R_{ab}+R_{bc}+R_{ca}}}{\frac{R_{ca}R_{bc}}{R_{ab}+R_{bc}+R_{ca}}}$$

$$\frac{R_A R_B}{R_C} = \frac{(R_{ab})^2}{(R_{ab}+R_{bc}+R_{ca})}$$

$$\frac{R_A R_B}{R_C} = \frac{(R_{ab})^2 R_{bc}}{(R_{ab}+R_{bc}+R_{ca})R_{bc}}$$

$$\frac{R_A R_B}{R_C} = \frac{R_{ab}}{R_{bc}} * R_B$$

$$\boxed{\frac{R_A}{R_C} = \frac{R_{ab}}{R_{bc}}}$$

In the same way

$$\boxed{\frac{R_B}{R_C} = \frac{R_{ab}}{R_{ac}}}$$

$$R_{ab} = \frac{R_B}{R_C} R_{ac} \qquad (1.18)$$

$$\boxed{\frac{R_A}{R_B} = \frac{R_{ac}}{R_{bc}}}$$

$$R_{ac} = \frac{R_A}{R_B} R_{bc} \qquad (1.19)$$

Substitute (1.18) & (1.19) in (1.15)

$$R_A = \frac{\frac{R_B}{R_C} R_{ac} * \frac{R_A}{R_B} R_{bc}}{\frac{R_B}{R_C} R_{ac} + R_{bc} + \frac{R_A}{R_B} R_{bc}}$$

$$R_A = \frac{\frac{R_A}{R_C} R_{bc}}{\frac{R_B}{R_C} + 1 + \frac{R_A}{R_B}}$$

After solving above equation

$$R_{bc} = R_B + R_C + \frac{R_B R_C}{R_A}$$

$$\boxed{R_{bc} = R_B + R_C + \frac{R_B R_C}{R_A}}$$

In the same way,

$$\boxed{R_{ab} = R_A + R_B + \frac{R_A R_B}{R_C}}$$

$$\boxed{R_{ac} = R_A + R_C + \frac{R_A R_C}{R_B}}$$

Nodal and Mesh Analysis (Nodal Voltage and Mesh Current Analysis)

The simple series & parallel circuits can be solved by using ohm`s law & Kirchhoff's law.

If the circuits are complex, conducting several sources & a large number of elements, they may be simplified using star-delta transformation. There are also other effective solving complex electric circuits.

Mesh current or loop current analysis & node voltage analysis are the two very effective methods of solving complex electric circuits. We have various network theorems which are also effective alternate methods to solve complex electrical circuits.

1. Node voltage analysis
2. Mesh current or loop current analysis

Nodal Analysis (KCL + ohm's Law)

In Nodal analysis, we will apply Kirchhoff's current law to determine the potential difference (voltage) at any node with respect to some arbitrary reference point in a network. Once the potentials of all nodes are known, it is a simple matter to determine other quantities such as current and power within the network.

Simple steps:

1. Identify the Number of nodes when current is dividing and assign voltage to nodes.
2. Write KCL equation at each node and except as reference node.
3. Write ohm's law form for current in nodal equation &solve the equation.

In other way the steps used in solving a circuit using Nodal analysis are explained elaborately as follows:

i. Arbitrarily assign a reference node within the circuit and indicate this node as ground. The reference node is usually located at the bottom of the circuit, although it may be located anywhere.
ii. Convert each voltage source in the network to its equivalent current source. This step, although not absolutely necessary, makes further calculations easier to understand.
iii. Arbitrarily assign voltages $(V_1, V_2, \ldots V_n)$ to the remaining nodes in the circuit. (Remember that you have already assigned a reference node, so these voltages will all be with respect to the chosen reference.)
iv. Arbitrarily assign a current direction to each branch in which there is no current source. Using the assigned current directions, indicate the corresponding polarities of the voltage drops on all resistors.
v. With the exception of the reference node (ground), apply Kirchhoff's current law at each of the nodes. If a circuit has a total of n+1 nodes (including the reference node), there will be n simultaneous linear equations.
vi. Rewrite each of the arbitrarily assigned currents in terms of the potential difference across a known resistance.
vii. Solve the resulting simultaneous linear equations for the voltages $(V_1, V_2, \ldots V_n)$.

Mesh analysis (KVL + ohm's Law)

The Mesh Current Method, also known as the **Loop Current** Method, is quite similar to the Branch Current method in that it uses simultaneous equations, Kirchhoff's Voltage Law, Kirchhoff's Current Law and Ohm's Law to determine unknown currents in a network. It differs from the Branch Current method in that it does not use Kirchhoff's Current Law, and it is usually able to solve a circuit with less unknown variables and less simultaneous equations, which is especially nice if you're forced to solve without a calculator.

A mesh is a loop which does not contain any other loops within it.

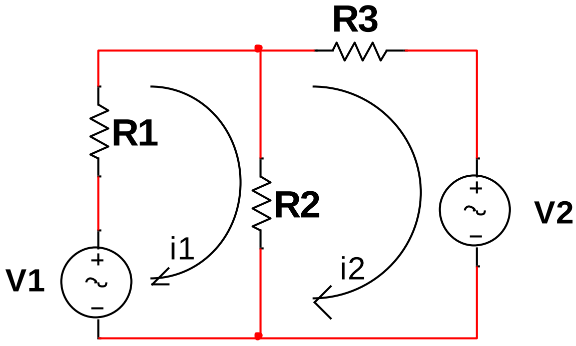
Simple Steps:

1. Identify the Number of Loops/ Meshes.

2. Assign the currents in each loop.
3. Apply KVL for each mesh and write ohm's law form.
4. Solve the equations and obtain mesh currents.

Step 1: Number of loops is identified as 2.

Step 2: Currents i_1 and i_2 are assigned to each loop in clockwise direction.



Step 3:

Apply KVL for loop (1)

$$-V_1 + i_1R_1 + (i_1 - i_2)R_2 = 0$$

$$i_1(R_1 + R_2) - i_2R_2 = V_1 \quad (1.20)$$

Apply KVL for loop (2)

$$V_2 + i_2R_3 + (i_2 - i_1)R_2 = 0$$

$$-i_1R_2 + i_2(R_2 + R_3) = -V_2 \quad (1.21)$$

Step 4: Solve the equations 1.20 and1.21 we get the loop currents i_1 and i_2. From loop currents we can calculate the branch currents also.

In other way the steps used in solving a circuit using mesh analysis are explained elaborately as follows:

i. Arbitrarily assign a clockwise current to each interior closed loop in the network. Although the assigned current may be in any direction, a clockwise direction is used to make later work simpler.

ii. Using the assigned loop currents, indicate the voltage polarities across all resistors in the circuit. For a resistor that is common to two loops, the polarities of the voltage drop due to each loop current should be indicated on the appropriate side of the component.

iii. Applying Kirchhoff's voltage law, write the loop equations for each loop in the network. Do not forget that resistors that are common to two loops will have two voltage drops, one due to each loop.

iv. Solve the resultant simultaneous linear equations.

v. Branch currents are determined by algebraically combining the loop currents that are common to the branch.

Nodal Versus Mesh Analysis

Both nodal and mesh analyses provide a systematic way of analyzing a complex network. The choice of the better method is dictated by two factors.

1. The first factor is the nature of the particular network. Networks that contain many series-connected elements, voltage sources, or super meshes are more suitable for mesh analysis, whereas networks with parallel-connected elements, current sources, or super nodes are more suitable for nodal analysis. Also, a circuit with fewer nodes than meshes is better analyzed using nodal analysis, while a circuit with fewer meshes than nodes is better analyzed using mesh analysis. The key is to select the method that results in the smaller number of equations.
2. The second factor is the information required. If node voltages are required, it may be expedient to apply nodal analysis. If branch or mesh currents are required, it may be better to use mesh analysis.

It is helpful to be familiar with both methods of analysis, for at least two reasons. First, one method can be used to check the results from the other method, if possible. Second, since each method has its limitations, only one method may be suitable for a particular problem.

Nodal Analysis with Voltage Sources

Now we see how voltage sources affect nodal analysis. We use the circuit in Fig. 1.4 for illustration. Consider the following two possibilities.

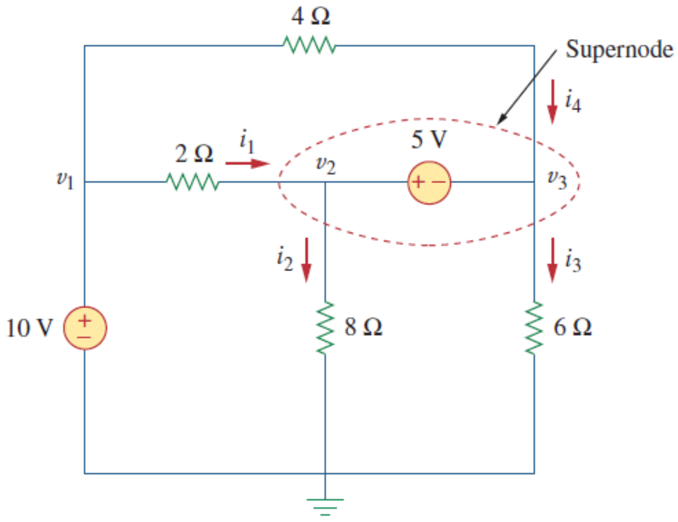


Figure1.4 A circuit with Super node

Case 1: If a voltage source is connected between the reference node and a non-reference node, we simply set the voltage at the non-reference node equal to the voltage of the voltage source. In Fig. 1. for example,

$$V_1 = 10V$$

Thus, our analysis is somewhat simplified by this knowledge of the voltage at this node.

Case 2 (Supernode): If the voltage source (dependent or independent) is connected between two non-reference nodes, the two non-reference nodes form a generalized node or super node; we apply both KCL and KVL to determine the node voltages.

In the above circuit nodes 2 and 3 form a super node. We analyze a circuit with super nodes using the same steps mentioned in the nodal analysis except that the super nodes are treated differently. Why because an essential component of nodal analysis is applying KCL, which requires knowing the current through each element. There is no way of knowing the current through a voltage source in advance. However, KCL must be satisfied at a super node like any other node.

KCL at Super node

$$i_1 + i_4 = i_2 + i_3$$

$$\frac{V_1 - V_2}{2} + \frac{V_1 - V_3}{4} = \frac{V_2 - 0}{8} + \frac{V_3 - 0}{6}$$

To apply Kirchhoff's voltage law to the super node were draw the circuit as shown in Fig. 1.5

KVL at Super node

$$-V_2 + 5 + V_3 = 0 \qquad \Rightarrow \qquad V_2 - V_3 = 5$$

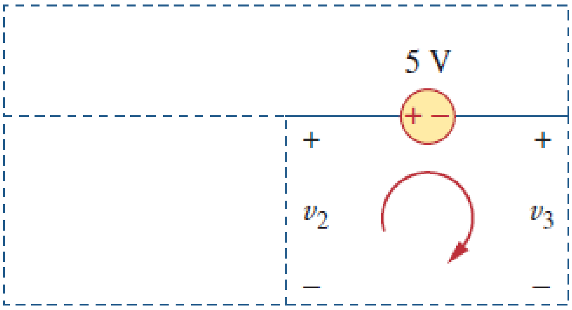


Fig.1.5 Applying KVL at super node

At remaining nodes (except Super node and reference node) apply KCL and solve all the equations we get the nodal voltages.

Properties of a super node:

1. The voltage source inside the super node provides a constraint equation needed to solve for the node voltages.
2. A super node has no voltage of its own.
3. A super node requires the application of both KCL and KVL.

Mesh Analysis with Current Sources

Applying mesh analysis to circuits containing current sources (dependent or independent) may appear complicated. But it is actually much easier because the presence of the current sources reduces the number of equations. Consider the following two possible cases.

CASE 1:

It is about current source exists only in one mesh. Consider the circuit in Fig. 1.6. It is clear that in mesh2

$$i_2 = -5A$$

Write a mesh equation for the mesh1

$$-10 + 4i_1 + 6(i_1 - i_2) = 0 \quad \Longrightarrow \quad i_1 = -2A$$

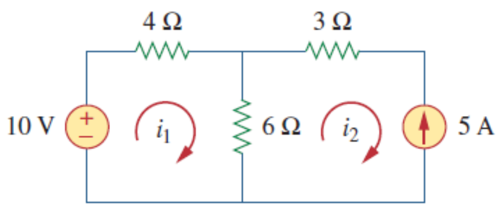
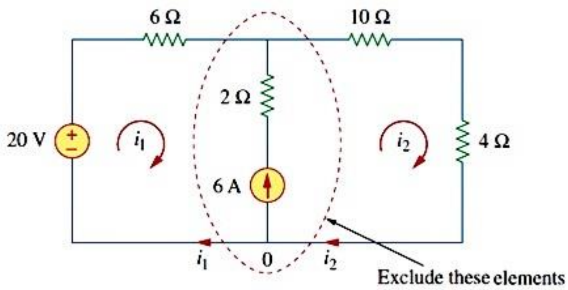


Fig.1.6 A circuit with current source

When a current source exists between two meshes: Consider the circuit in Fig. 1.7(a), for example. We create a super mesh by excluding the current source and any elements connected in series with it, as shown in Fig. 1(b). Thus,



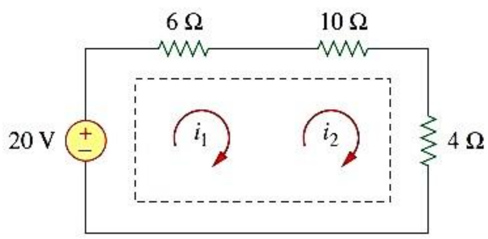


Figure1.7 (a) Two meshes having a current source in common

Figure 1.7(b) a super mesh, created by excluding the current source

As shown in Fig. 1.7(b), we create a super mesh as the periphery of the two meshes and treat it differently. (If a circuit has two or more super meshes that intersect, they should be combined to form a larger super mesh.) Why treat the super mesh differently? Because mesh analysis applies KVL—which requires that we know the voltage across each branch—and we do not know the voltage across a current source in advance. However, a super mesh must satisfy KVL like any other mesh. Therefore, applying KVL to the super mesh in Fig. 1.7(b) gives

$$-20 + 6i_1 + 10i_2 + 4i_2 = 0$$

$$6i_1 + 14i_2 = 20$$

We apply KCL to a node in the branch where the two meshes intersect.

Applying KCL to node 0 in Fig. 1.7(a) gives $i_2 = i_1 + 6$

Solve above two equations we get the solution as $i_1 = -3.2A, \quad i_2 = 2.8A$

ELEMENTS OF ELECTRICAL CIRCUITS

UNIT – III (Introduction to Single Phase AC Circuits)

Objectives:

- To understand the concepts of RMS, average values for periodic waves.
- To understand the concepts of phase and phase differences in AC circuits.
- To understand the complex and polar form representations of complex quantities and J notation.

Syllabus:

Generation of alternating sinusoidal quantities - R.M.S, Average values and form factor for different periodic wave forms – sinusoidal alternating quantities – Phase and Phase difference – Complex and polar forms of representations, J Notation.

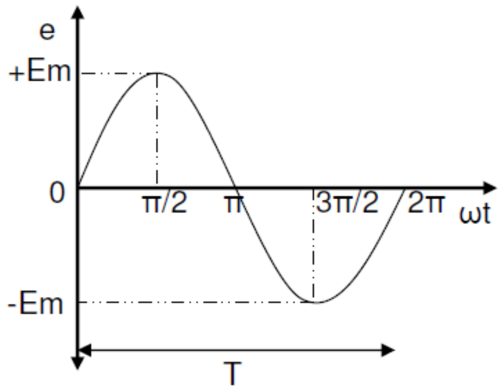
Outcomes:

On completion the student should be able to:

- Understand various terminologies such as average, RMS, form factor, peak factor used in AC circuits.
- Evaluate various parameters of alternating quantities.
- Analyse the phase and phase difference in AC quantities.

Learning Material

Definition of Alternating Quantity:



An alternating quantity changes continuously in magnitude and alternates in direction at regular intervals of time. Important terms associated with an alternating quantity are defined below.

1.Amplitude:

It is the maximum value attained by an alternating quantity. Also called as maximum or peak value.

2.Time Period (T):

It is the Time Taken in seconds to complete one cycle of an alternating quantity.

3. Instantaneous Value:

It is the value of the quantity at any instant.

4. Frequency (f):

It is the number of cycles that occur in one second. The unit for frequency is Hz or cycles/sec.The relationship between frequency and time period can be derived as follows.

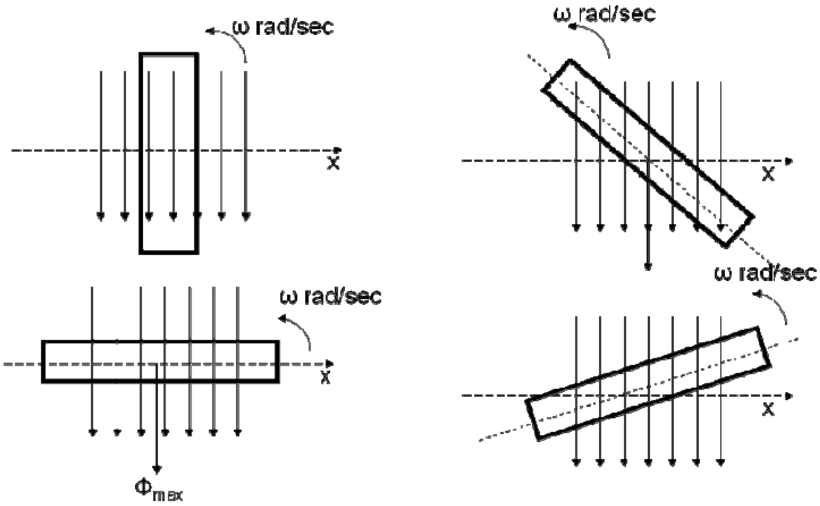
Time taken to complete f cycles = 1 second

Time taken to complete 1 cycle = 1/f second

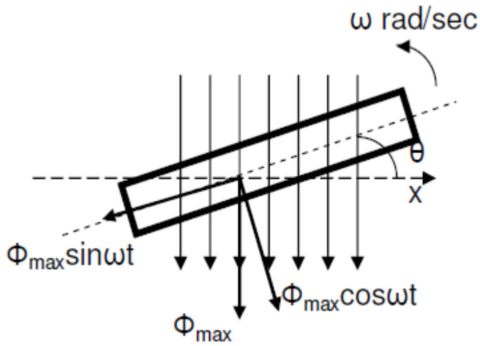
$$\mathbf{T = 1/f}$$

Generation of sinusoidal AC voltage:

Consider a rectangular coil of N turns placed in a uniform magnetic field as shown in the figure. The coil is rotating in the anticlockwise direction at an uniform angular velocity of rad/sec.



When the coil is in the vertical position, the flux linking the coil is zero because the plane of the coil is parallel to the direction of the magnetic field. Hence at this position, the emf induced in the coil is zero. When the coil moves by some angle in the anticlockwise direction, there is a rate of change of flux linking the coil and hence an emf is induced in the coil. When the coil reaches the horizontal position, the flux linking the coil is maximum, and hence the emf induced is also maximum. When the coil further moves in the anticlockwise direction, the emf induced in the coil reduces. Next when the coil comes to the vertical position, the emf induced becomes zero. After that the same cycle repeats and the emf is induced in the opposite direction. When the coil completes one complete revolution, one cycle of AC voltage is generated. The generation of sinusoidal AC voltage can also be explained using mathematical equations. Consider a rectangular coil of N turns placed in a uniform magnetic field in the position shown in the figure. The maximum flux linking the coil is in the downward direction as shown in the figure. This flux can be divided into two components, one component acting along the plane of the coil $\Phi_{max}\sin\omega t$ and another component acting perpendicular to the plane of the coil $\Phi_{max}\cos\omega t$.



The component of flux acting along the plane of the coil does not induce any flux in the coil. Only the component acting perpendicular to the plane of the coil i.e. $\Phi_{max}\cos\omega t$ induces an emf in the coil. Hence the emf induced in the coil is a sinusoidal emf. This will induce a sinusoidal current in the circuit given by

$$\emptyset = \phi_{max} cos\omega t$$

$$e = -N\frac{d\phi}{dt} = -N\frac{d}{dt}\phi_{max} cos\omega t = N\phi_{max}\omega sin\omega t = E_m sin\omega t$$

Hence the emf induced in the coil is a sinusoidal emf. This will induce a sinusoidal current in the circuit given by

$$i = I_m sin\omega t$$

Angular Frequency (ω):

Angular frequency is defined as the number of radians covered in one second(ie the angle covered by the rotating coil). The unit of angular frequency is rad/sec.

$$\omega = \frac{2\pi}{T} = 2\pi f$$

Average Value

The arithmetic average of all the values of an alternating quantity over one cycle is called its average value

$$Average\ Value = \frac{Area\ under\ one\ cycle}{Base}$$

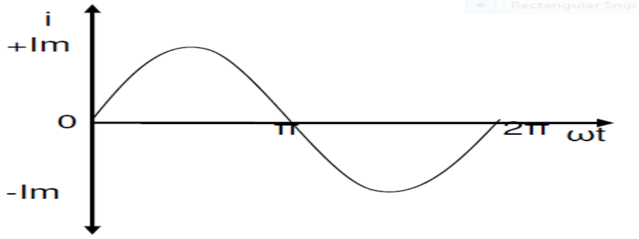
$$V_{avg} = \frac{1}{2\pi}\int_0^{2\pi} v\ d(\omega t)$$

For Symmetrical waveforms, the average value calculated over one cycle becomes equal to zero because the positive area cancels the negative area. Hence for symmetrical waveforms, the average value is calculated for half cycle.

$$Average\ Value = \frac{Area\ under\ one\ half\ cycle}{Base}$$

$$V_{avg} = \frac{1}{\pi}\int_0^{\pi} v\ d(\omega t)$$

Average value of a sinusoidal current

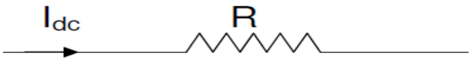


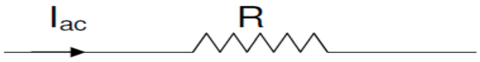
$$i = I_m sin\omega t$$

$$I_{avg} = \frac{1}{\pi}\int_0^{\pi} i d(\omega t) = \frac{1}{\pi}\int_0^{\pi} I_m\ sin\omega t\ d(\omega t) = \frac{2I_m}{\pi} = 0.637 I_m$$

RMS or Effective Value

The effective or RMS value of an alternating quantity is that steady current (dc) which when flowing through a given resistance for a given time produces the same amount of heat produced by the alternating current flowing through the same resistance for the same time.

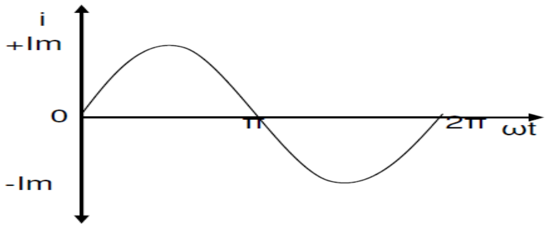




$$RMS = \sqrt{\frac{Area\ under\ squared\ curve}{Base}}$$

$$V_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} v^2 d(\omega t)}$$

RMS value of a sinusoidal current



$$i = I_m sin\omega t \,, I_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} i^2 d(\omega t)} = \sqrt{\frac{1}{\pi}\int_0^{\pi} {I_m}^2 \, sin^2\omega t \, d(\omega t)} = \frac{I_m}{\sqrt{2}} = 0.707 I_m$$

Form Factor:

The ratio of RMS value to the average value of an alternating quantity is known as Form Factor

$$Form\ Factor = \frac{RMS\ Value}{Average\ Value}$$

Peak Factor or Crest Factor:

The ratio of maximum value to the RMS value of an alternating quantity is known as the peak factor

$$Peak\ Factor = \frac{Maximum\ Value}{RMS\ Value}$$

For a sinusoidal waveform

$$I_{avg} = \frac{2I_m}{\pi} = 0.637 I_m$$

$$I_{rms} = \frac{I_m}{\sqrt{2}} = 0.707 I_m$$

$$FF = \frac{I_{rms}}{I_{avg}} = 1.11$$

$$PF = \frac{I_m}{I_{rms}} = 1.414$$

Phasor Representation:

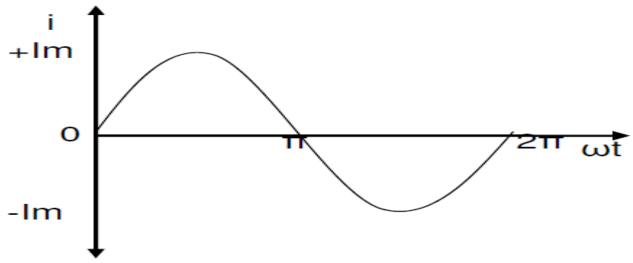
An alternating quantity can be represented using

(i) Waveform

(ii) Equations

(iii) Phasor

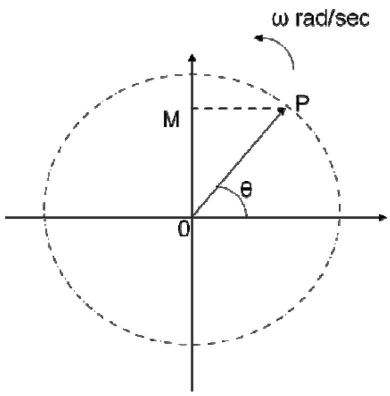
A sinusoidal alternating quantity can be represented by a rotating line called a **Phasor.** A phasor is a line of definite length rotating in anticlockwise direction at a constant angular velocity. The waveform and equation representation of an alternating current is as shown. This sinusoidal quantity can also be represented using phasors.



$I = i_m \sin\omega t$

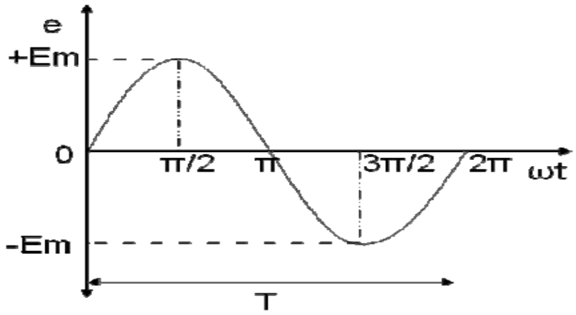
Draw a line OP of length equal to Im. This line OP rotates in the anticlockwise direction with an uniform angular velocity **ω** rad/sec and follows the circular trajectory shown in figure. Atany instant, the projection of OP on the y-axis is given by OM=OPsinθ =. **$I = i_m \sin\omega t$**

Hence the line OP is the phasor representation of the sinusoidal current



Phase:

Phase is defined as the fractional part of time period or cycle through which the quantity has advanced from the selected zero position of reference.

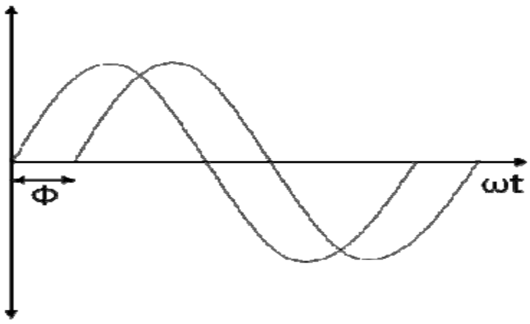


Phase of $+E_m$ is $\pi/2$ rad or T/4 sec

Phase of $-E_m$ is $3\pi/2$ rad or 3T/4 sec

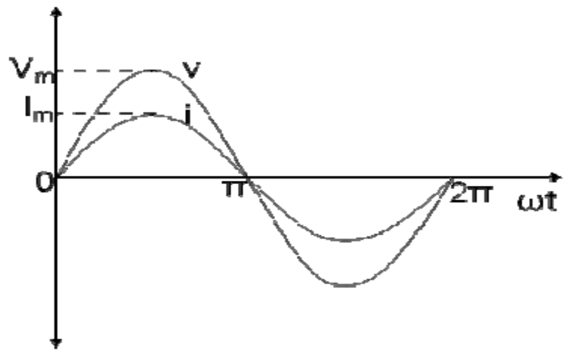
Phase Difference:

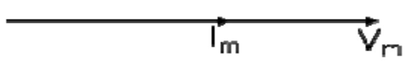
When two alternating quantities of the same frequency have different zero points, they are said to have a phase difference. The angle between the zero points is the angle of phase difference.



In Phase

Two waveforms are said to be in phase, when the phase difference between them is zero. That is the zero points of both the waveforms are same. The waveform, phasor and equation representation of two sinusoidal quantities which are in phase is as shown. The figure shows that the voltage and current are in phase.



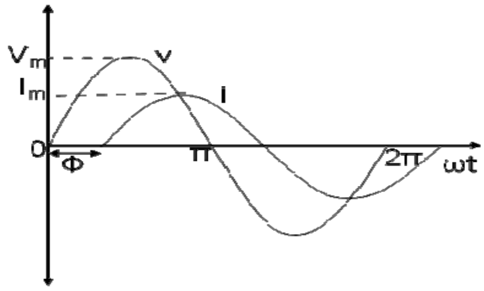


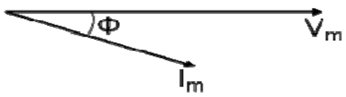
$V = V_m \sin\omega t$

$I = I_m \sin\omega t$

Lagging:

In the figure shown, the zero point of the current waveform is after the zero point of the voltage waveform. Hence the current is lagging behind the voltage. The waveform, phasor and equation representation is as shown.



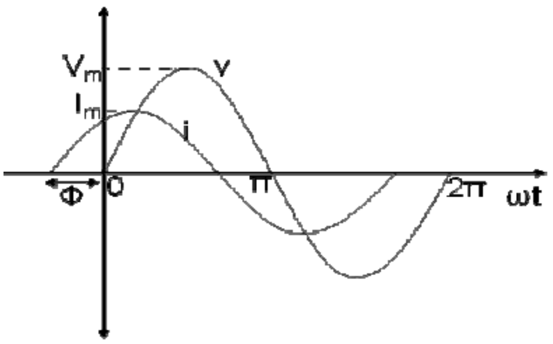


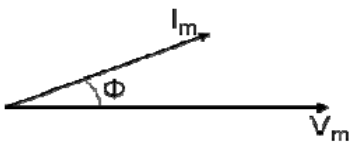
$V = V_m \sin\omega t$

$I = I_m \sin(\omega t - \Phi)$

Leading

In the figure shown, the zero point of the current waveform is before the zero point of the voltage waveform. Hence the current is leading the voltage. The waveform, phasor and equation representation is as shown.





$V = V_m \sin\omega t$

$I = I_m \sin(\omega t + \Phi)$

Complex numbers:

The mathematics used in Electrical Engineering to add together resistances, currents or DC voltages uses what are called "real numbers". But real numbers are not the only kind of numbers we need to use especially when dealing with frequency dependent sinusoidal sources and vectors. As well as using normal or real numbers, **Complex Numbers** were introduced to allow complex equations to be solved with numbers that are the square roots of negative numbers, $\sqrt{-1}$.

In electrical engineering this type of number is called an "imaginary number" and to distinguish an imaginary number from a real number the letter " j " known commonly in electrical engineering as the **j-operator**, is used. The letter j is placed in front of a real number to signify its imaginary number operation. Examples of imaginary numbers are: j3, j12, j100 etc. Then a **complex number** consists of two distinct but very much related parts, a " Real Number " plus an " Imaginary Number ".

Complex Numbers represent points in a two dimensional complex or s-plane that are referenced to two distinct axes. The horizontal axis is called the "real axis" while the vertical axis is called the "imaginary axis". The real and imaginary parts of a complex number, Z are abbreviated as Re(z) and Im(z), respectively.

Complex numbers that are made up of real (the active component) and imaginary (the reactive component) numbers can be added, subtracted and used in exactly the same way as elementary algebra is used to analyse **DC Circuits**.

The rules and laws used in mathematics for the addition or subtraction of imaginary numbers are the same as for real numbers, j2 + j4 = j6 etc. The only difference is in multiplication because two imaginary numbers multiplied together becomes a positive real number, as two negatives make a positive. Real numbers can also be thought of as a complex number but with a zero imaginary part labelled j0.

The **j-operator** has a value exactly equal to $\sqrt{-1}$, so successive multiplication of " j ", (j x j) will result in j having the following values of, -1, -j and +1. As the j-operator is commonly used to indicate the anticlockwise rotation of a vector, each successive multiplication or power of " j ", j^2, j^3 etc, will force the vector to rotate through an angle of 90° anticlockwise as shown below. Likewise, if the multiplication of the vector results in a -j operator then the phase shift will be -90°, i.e. a clockwise rotation.

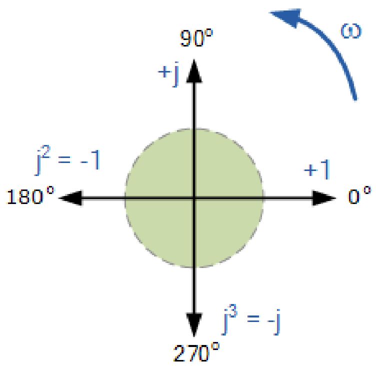
Vector Rotation of the j-operator:

$$90^{\circ} \text{ rotation: } \quad j^1 = \sqrt{-1} = +j$$

$$180^{\circ} \text{ rotation: } \quad j^2 = \left(\sqrt{-1}\right)^2 = -1$$

$$270^{\circ} \text{ rotation: } \quad j^3 = \left(\sqrt{-1}\right)^3 = -j$$

$$360^{\circ} \text{ rotation: } \quad j^4 = \left(\sqrt{-1}\right)^4 = +1$$



So by multiplying an imaginary number by j^2 will rotate the vector by 180° anticlockwise, multiplying by j^3 rotates it 270° and by j^4 rotates it 360° or back to its original position. Multiplication by j^{10} or by j^{30} will cause the vector to rotate anticlockwise by the appropriate amount. In each successive rotation, the magnitude of the vector always remains the same.

In Electrical Engineering there are different ways to represent a complex number either graphically or mathematically. One such way that uses the cosine and sine rule is called the **Cartesian** or **Rectangular Form**.

Complex Numbers using the Rectangular Form:

In the last tutorial about **Phasors**, we saw that a complex number is represented by a real part and an imaginary part that takes the generalised form of: Z= x + jy

Where:

- Z - is the Complex Number representing the Vector
- x - is the Real part or the Active component
- y - is the Imaginary part or the Reactive component
- j - is defined by $\sqrt{-1}$

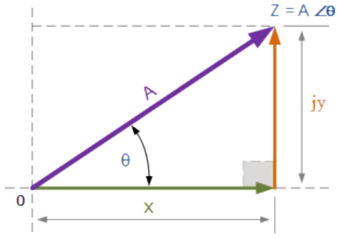
In the rectangular form, a complex number can be represented as a point on a two-dimensional plane called the **complex** or **s-plane**. So for example, Z = 6 + j4 represents a single point whose coordinates represent 6 on the horizontal real axis and 4 on the vertical imaginary axis.

Complex Numbers using Polar Form:

Unlike rectangular form which plots points in the complex plane, the **Polar Form** of a complex number is written in terms of its magnitude and angle. Thus, a polar form vector is presented as: $Z = A \angle \pm\theta$, where: Z is the complex number in polar form, A is the magnitude or modulo of the vector and θ is its angle or argument of A which can be either positive or negative. The magnitude and angle of the point still remains the same as for the rectangular form above, this time in polar form the location of the point is represented in a "triangular form" as shown below.

Polar Form Representation of a Complex Number:

As the polar representation of a point is based around the triangular form, we can use simple geometry of the triangle and especially trigonometry and Pythagoras's Theorem on triangles to find both the magnitude and the angle of the complex number. As we remember from school, trigonometry deals with the relationship between the sides and the angles of triangles so we can describe the relationships between the sides as:



$$A^2 = x^2 + y^2$$

$$A = \sqrt{\mathbf{x^2 + y^2}}\ \boldsymbol{\theta}$$

$$x = A\cos\boldsymbol{\theta},\ y = A\sin\boldsymbol{\theta}$$

Using trigonometry again, the angle θ of A is given as follows.

$$\theta = tan^{-1}\frac{y}{x}$$

Then in Polar form the length of A and its angle represents the complex number instead of a point. Also in polar form, the conjugate of the complex number has the same magnitude or modulus it is the sign of the angle that changes, so for example the conjugate of $6\angle 30^o$ would be $6\angle -30^o$

Converting between Rectangular Form and Polar Form:

In the rectangular form we can express a vector in terms of its rectangular coordinates, with the horizontal axis being its real axis and the vertical axis being its imaginary axis or j-component. In polar form these real and imaginary axes are simply represented by "$A\angle\theta$".

ELEMENTS OF ELECTRICAL CIRCUITS

UNIT – IV (Sinusoidal Steady State Analysis)

Objectives:

- To analyse series parallel combinations of R, L, C in steady state.
- To understand the concepts of complex power and power factor.

Syllabus:

Steady state analysis of R, L and C (in series, parallel and series parallel combinations) with sinusoidal excitation-Concept of Reactance, Impedance, Susceptance and Admittance-Power Factor and significance of Real and Reactive power, Complex Power.

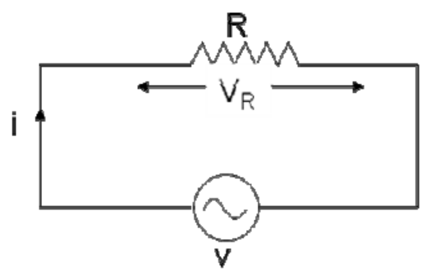
Outcomes:

On completion the student should be able to:

- Analyse various series parallel combinations of R, L, C in steady state.

Learning Material

AC circuit with a pure resistance



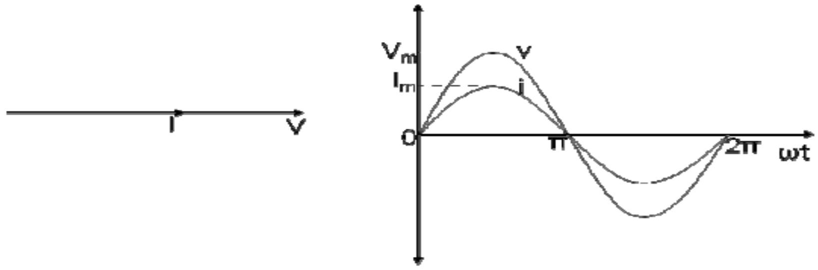
Consider an AC circuit with a pure resistance R as shown in the figure. The alternating voltage v is given by

$V = V_m \, Sin\omega t$

Using ohms law, we can write the following relations

$$i = \frac{V}{R} = \frac{V_m sin\omega t}{R} = I_m sin\omega t \, , where \; I_m = \frac{V_m}{R}$$

From equation (1) and (2) we conclude that in a pure resistive circuit, the voltage and current are in phase. Hence the voltage and current waveforms and phasors can be drawn as below



Instantaneous power

The instantaneous power in the above circuit can be derived as follows:

$$p = vi$$

$$p = (V_m sin\omega t)(I_m sin\omega t\,) = V_m I_m sin^2\omega t = \frac{V_m I_m}{2}(1 - cos2\omega t) = \frac{V_m I_m}{2} - \frac{V_m I_m}{2}cos2\omega t$$

The instantaneous power consists of two terms. The first term is called as the constant power term and the second term is called as the fluctuating power term.

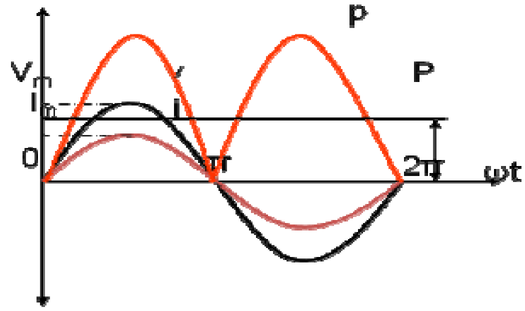
Average power

From the instantaneous power we can find the average power over one cycle as follows

$$P = \frac{1}{2\pi}\int_0^{2\pi}\left[\frac{V_m I_m}{2} - \frac{V_m I_m}{2}cos2\omega t\right]d\omega t = \frac{V_m I_m}{2} - \frac{1}{2\pi}\int_0^{2\pi}\left[\frac{V_m I_m}{2}cos2\omega t\right]d\omega t = \frac{V_m I_m}{2}$$

$$P = \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}} = V.I$$

As seen above the average power is the product of the rms voltage and the rms current.The voltage, current and power waveforms of a purely resistive circuit is as shown in figure.

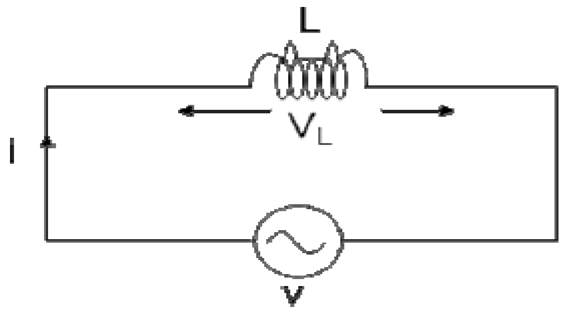


As seen from the waveform, the instantaneous power is always positive meaning that the power always flows from the source to the load.

Phasor Algebra for a pure resistive circuit

$$\bar{V} = V\angle 0^0 = V + j0, \bar{I} = \frac{\bar{V}}{R} = \frac{V + j0}{R} = I + j0 = I\angle 0^0$$

AC circuit with a pure inductance



Consider an AC circuit with a pure inductance L as shown in the figure. The alternating voltage v is given by

$$V=V_m \; Sin\omega t \text{ -------------(1)}$$

The current flowing in the circuit is i. The voltage across the inductor is given as V_L which is the same as v.

We can find the current through the inductor as follows

$$v = L\,\frac{di}{dt}$$

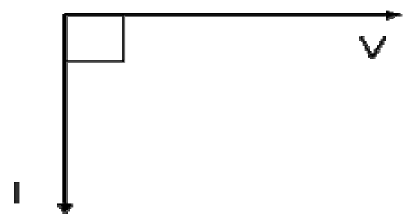
$$V_m sin\omega t = L\,\frac{di}{dt}$$

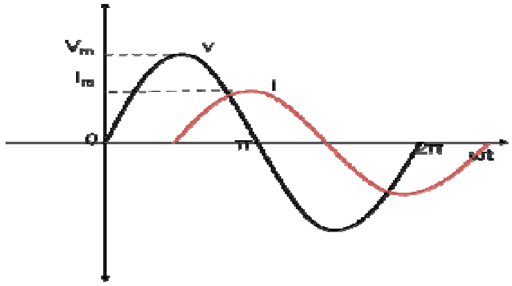
$$di = \frac{V_m}{L} sin\omega t$$

$$i = \frac{V_m}{L}\int sin\omega t\,dt = \frac{V_m}{\omega L}\,(-cos\,\omega t\,) = \frac{V_m}{\omega L}\,sin(\omega t - {}^{\pi}/_{2}\,) = \;I_m\,sin(\omega t - {}^{\pi}/_{2}\,)$$

where $I_m = \frac{V_m}{\omega L}$

From equation (1) and (2) we observe that in a pure inductive circuit, the current lags behind the voltage by 90^0. Hence the voltage and current waveforms and phasors can be drawn as below.





Inductive reactance

The inductive reactance X_L is given as

$$X_L = \omega L = 2\pi f\,L$$

$$I_m = \frac{V_m}{X_L}$$

It is equivalent to resistance in a resistive circuit. The unit is ohms (Ω)

Instantaneous power

The instantaneous power in the above circuit can be derived as follows

$$p = vi = (V_m sin\omega t)\big(I_m \sin(\omega t - {}^{\pi}/_{2})\big) = -V_m I_m sin\omega t\; cos\omega t = -\frac{V_m I_m}{2} sin2\omega t$$

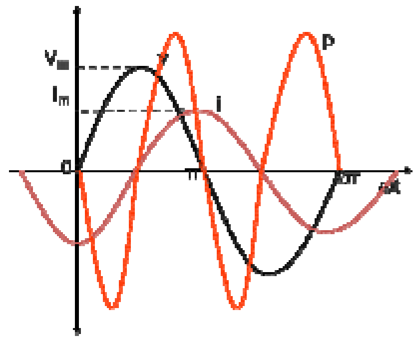
As seen from the above equation, the instantaneous power is fluctuating in nature.

Average power

From the instantaneous power we can find the average power over one cycle as follows

$$P = \frac{1}{2\pi}\int_0^{2\pi} -\frac{V_m I_m}{2} sin2\omega t\; d\;\; \omega t = 0$$

The average power in a pure inductive circuit is zero. Or in other words, the power consumed by a pure inductance is zero. The voltage, current and power waveforms of a purely inductive circuit is as shown in the figure.

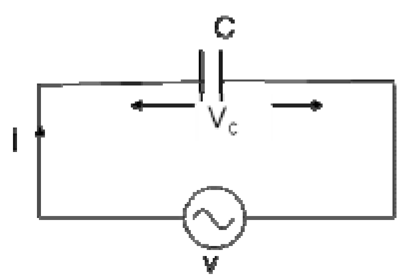


As seen from the power waveform, the instantaneous power is alternately positive and negative.When the power is positive, the power flows from the source to the inductor and when the power in negative, the power flows from the inductor to the source. The positive power is equal to the negative power and hence the average power in the circuit is equal to zero. The power just flows between the source and the inductor, but the inductor does not consume any power.

Phasor algebra for a pure inductive circuit:

$$\bar{V} = V + j0 = V\angle 0, \bar{Z} = jX_L = X_L\angle 90^0$$
$$\bar{I} = I\angle - 90^0 = 0 - jI$$
$$\bar{V} = \bar{I}(jX_L)$$

AC circuit with a pure capacitance:



Consider an AC circuit with a pure capacitance C as shown in the figure. The alternating voltage v is given by

$$V = V_m \, Sin\omega t \text{ -------------(1)}$$

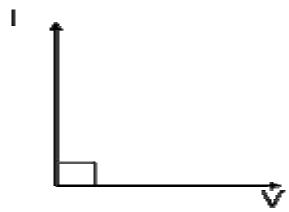
The current flowing in the circuit is i. The voltage across the capacitor is given as VC which is the same as v.We can find the current through the capacitor as follows

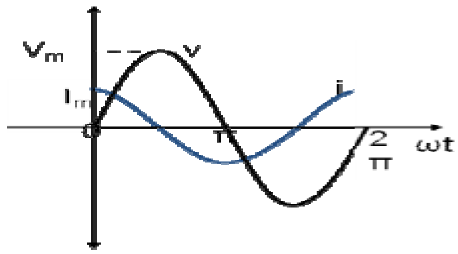
$$q = Cv = CV_m sin\omega t$$

$i = \frac{dq}{dt} = CV_m\omega cos\omega t = \omega CV_m \sin\left(\omega t + {}^{\pi}/_{2}\right) = I_m \sin\left(\omega t + {}^{\pi}/_{2}\right)$---(2)

Where $I_m = \omega CV_m$

From equation (1) and (2) we observe that in a pure capacitive circuit, the current leads the voltage by 90^0. Hence the voltage and current waveforms and phasors can be drawn as below.





$$p = vi$$
$$p = (V_m \sin \omega t)(I_m \sin(\omega t + \pi/2))$$
$$p = V_m I_m \sin \omega t \cos \omega t$$
$$p = \frac{V_m I_m}{2} \sin 2\omega t$$

Capacitive reactance

The capacitive reactance X_C is given as

$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$, $I_m = \frac{V_m}{X_C}$

It is equivalent to resistance in a resistive circuit. The unit is ohms (Ω)

$$P = \frac{1}{2\pi} \int_0^{2\pi} \frac{V_m I_m}{2} \sin 2\omega t \, d\omega t$$
$$P = 0$$

Instantaneous power

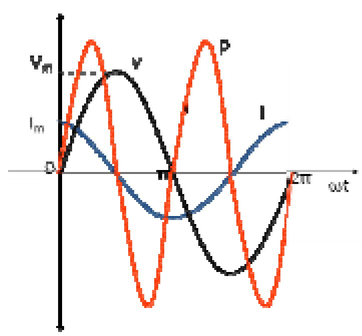
The instantaneous power in the above circuit can be derived as follows

As seen from the above equation, the instantaneous power is fluctuating in nature.

Average power

From the instantaneous power we can find the average power over one cycle as follows

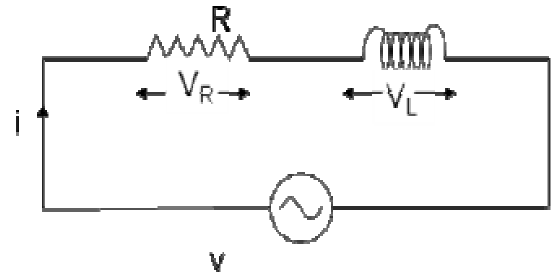
The average power in a pure capacitive circuit is zero. Or in other words, the power consumed by a pure capacitance is zero. The voltage, current and power waveforms of a purely capacitive circuit is as shown in the figure.



As seen from the power waveform, the instantaneous power is alternately positive and negative. When the power is positive, the power flows from the source to the capacitor and when the power in negative, the power flows from the capacitor to the source. The positive power is equal to the negative power and hence the average power in the circuit is equal to zero. The power just flows between the source and the capacitor, but the capacitor does not consume any power.

STEADY STATE ANALYSIS OF R,L,C, ELEMENTS WITH SINUSOIDAL EXCITATION

R-L Series circuit:



Consider an AC circuit with a resistance R and an inductance L connected in series as shown in the figure. The alternating voltage v is given by

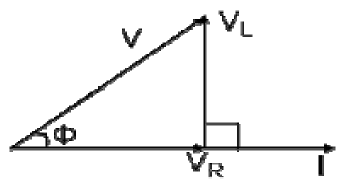
$$V = V_m \sin\omega t$$

The current flowing in the circuit is i. The voltage across the resistor is V_R and that across the inductor is V_L.

$V_R = IR$ is in phase with I

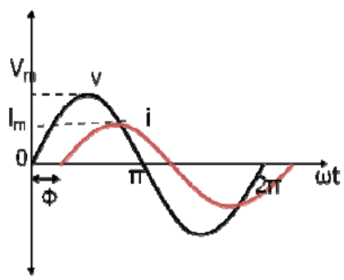
$V_L = IX_L$ leads current by 90 degrees

With the above information, the phasor diagram can be drawn as shown.



The current I is taken as the reference phasor. The voltage V_R is in phase with I and the voltage V_L leads the current by 90^0. The resultant voltage V can be drawn as shown in the figure. From the phasor diagram we observe that the voltage leads the current by an angle Φ or in other words the current lags behind the voltage by an angle Φ.

The waveform and equations for an RL series circuit can be drawn as below.



$$V = V_m \sin\omega t$$
$$I = I_m \sin(\omega t - \Phi)$$

From the phasor diagram, the expressions for the resultant voltage V and the angle Φ can be derived as follows.

$$V = \sqrt{V_R^2 + V_L^2}, V_R = IR, V_L = IX_L$$

$$V = \sqrt{(IR)^2 + (IX_L)^2} = I\sqrt{R^2 + {X_L}^2} = IZ$$

Where $Z = \sqrt{R^2 + {X_L}^2}$

The impedance in an AC circuit is similar to a resistance in a DC circuit. The unit for impedance is ohms (Ω).

Phase angle

$\Phi = \tan^{-1}\left(\frac{V_L}{V_R}\right)$ $\Phi = \tan^{-1}\left(\frac{\omega L}{R}\right)$

Instantaneous power

The instantaneous power in an RL series circuit can be derived as follows:

$$p = vi$$

$$p = (V_m \sin \omega t)(I_m \sin(\omega t - \Phi))$$

$$p = \frac{V_m I_m}{2}\cos\Phi - \frac{V_m I_m}{2}\cos(2\omega t - \Phi)$$

The instantaneous power consists of two terms. The first term is called as the constant power term and the second term is called as the fluctuating power term.

Average power

From the instantaneous power we can find the average power over one cycle as follows

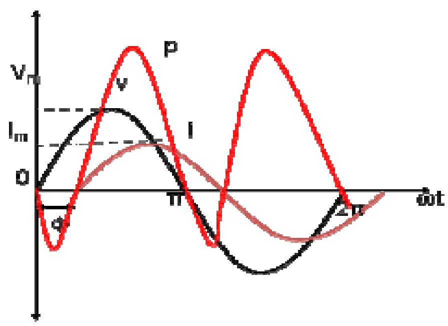
$$P = \frac{1}{2\pi}\int_0^{2\pi}\left[\frac{V_m I_m}{2}\cos\Phi - \frac{V_m I_m}{2}\cos(2\omega t - \Phi)\right]d\omega t$$

$$P = \frac{V_m I_m}{2}\cos\Phi$$

$$P = \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}\cos\Phi$$

$$P = VI\cos\Phi$$

The voltage, current and power waveforms of a RL series circuit is as shown in the figure.



As seen from the power waveform, the instantaneous power is alternately positive and negative. When the power is positive, the power flows from the source to the load and when the power in negative, the power flows from the load to the source. The positive power is not equal to the negative power and hence the average power in the circuit is not equal to zero.

From the phasor diagram,

$P = VI\cos\Phi$ $P = (IR*I)\cos\Phi$ $P = (I^2R)$

Hence the power in an RL series circuit is consumed only in the resistance. The inductance does not consume any power.

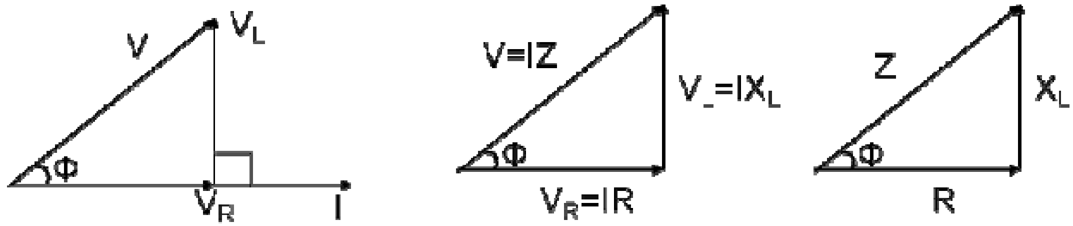
Power Factor

The power factor in an AC circuit is defined as the cosine of the angle between voltage and current i.e cos Φ

The power in an AC circuit is equal to the product of voltage, current and power factor

Impedance Triangle

We can derive a triangle called the impedance triangle from the phasor diagram of an RL series circuit as shown



The impedance triangle is right angled triangle with R and XL as two sides and impedance as the hypotenuse. The angle between the base and hypotenuse is Φ. The impedance triangle enables us to calculate the following things.

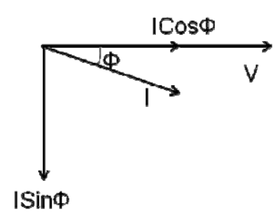
1. Impedance $Z = \sqrt{R^2 + X_L^2}$
2. Power Factor $\cos\Phi = \dfrac{R}{Z}$
3. Phase angle $\Phi = \tan^{-1}\left(\dfrac{X_L}{R}\right)$
4. Whether current leads or lags behind the voltage

Power

In an AC circuit, the various powers can be classified as

1. Real or Active power
2. Reactive power
3. Apparent power

Real or active power in an AC circuit is the power that does useful work in the cicuit. Reactive power flows in an AC circuit but does not do any useful work. Apparent power is the total power in an AC circuit.



From the phasor diagram of an RL series circuit, the current can be divided into two components. One component along the voltage IcosΦ, that is called as the active component of current and another component perpendicular to the voltage IsinΦ that is called as the reactive component of current.

Real Power

The power due to the active component of current is called as the active power or real power. It is denoted by P.

P = V x ICosΦ = I^2R

Real power is the power that does useful power. It is the power that is consumed by the resistance. The unit for real power in Watt(W).

Reactive Power

The power due to the reactive component of current is called as the reactive power. It is denoted by Q.

Q = V x ISinΦ = I^2X_L

Reactive power does not do any useful work. It is the circulating power in th L and C components. The unit for reactive power is Volt Amperes Reactive (VAR).

Apparent Power

The apparent power is the total power in the circuit. It is denoted by S.

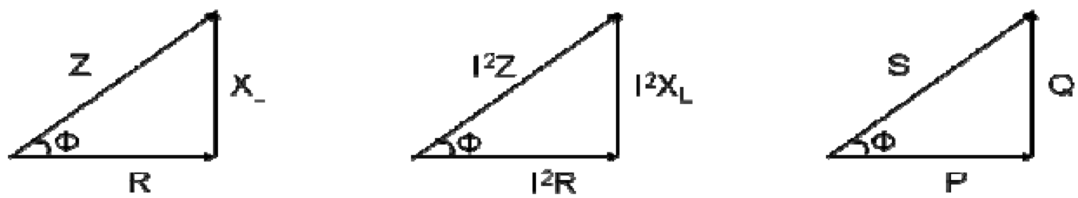
S = V x I = I^2Z

$$S = \sqrt{P^2 + Q^2}$$

The unit for apparent power is Volt Amperes (VA).

Power Triangle

From the impedance triangle, another triangle called the power triangle can be derived as shown.



The power triangle is right angled triangle with P and Q as two sides and S as the hypotenuse. The angle between the base and hypotenuse is Φ. The power triangle enables us to calculate the following things**.**

1. Apparent power $S = \sqrt{P^2 + Q^2}$
2. Power Factor $Cos\Phi = \frac{P}{S} = \frac{\mathrm{Re}\,alPower}{ApparentPower}$

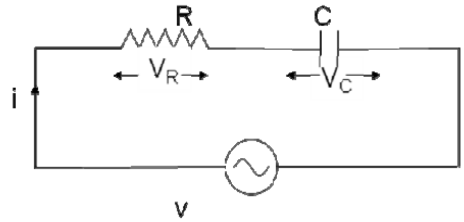
The power Factor in an AC circuit can be calculated by any one of the following methods

1. Cosine of angle between V and I
2. Resistance/Impedance R/Z
3. Real Power/Apparent Power P/S

Phasor algebra in a RL series circuit

$$V = V + j0 = V\angle 0, \bar{Z} = R + jX_L = Z\angle\phi, \bar{I} = \frac{\bar{V}}{\bar{Z}} = \frac{V}{Z}\angle -\phi\,, \qquad \bar{S} = VI^* = P + jQ$$

R-C Series circuit:



Consider an AC circuit with a resistance R and a capacitance C connected in series as shown in the figure. The alternating voltage v is given by

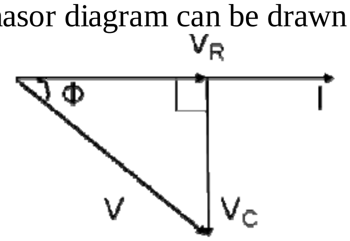
$$V = V_m \, Sin\omega t$$

The current flowing in the circuit is i. The voltage across the resistor is V_R and that across the capacitor is V_C.

V_R=IR is in phase with I

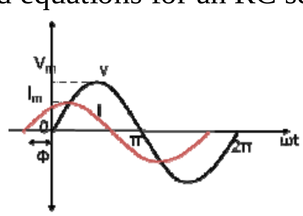
V_C=IX_C lags behind the current by 90 degrees

With the above information, the phasor diagram can be drawn as shown.



The current I is taken as the reference phasor. The voltage V_R is in phase with I and the voltage V_C lags behind the current by 90^0. The resultant voltage V can be drawn as shown in the figure. From the phasor diagram we observe that the voltage lags behind the current by an angle Φ or in other words the current leads the voltage by an angle Φ.

The waveform and equations for an RC series circuit can be drawn as below.



$$V = V_m \sin \omega t$$

$$I = I_m \sin(\omega t + \Phi)$$

From the phasor diagram, the expressions for the resultant voltage V and the angle _ can be derived as follows.

$$V = \sqrt{V_R^2 + {V_C}^2} = \sqrt{(IR)^2 + {IX_C}^2} = I\sqrt{(R)^2 + (X_C)^2} = IZ$$

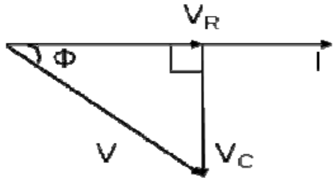
Where impedance $Z = \sqrt{(R)^2 + X_C)^2}$

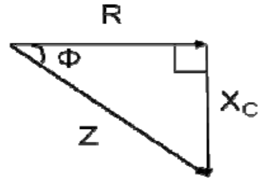
Phase angle $\phi = tan^{-1}\left(\frac{X_C}{R}\right) = tan^{-1}\left(\frac{1}{\omega RC}\right)$

Average power:

$$P = VIcos\emptyset = (IZ).I.\frac{R}{Z}$$

Hence the power in an RC series circuit is consumed only in the resistance. The capacitance does not consume any power.

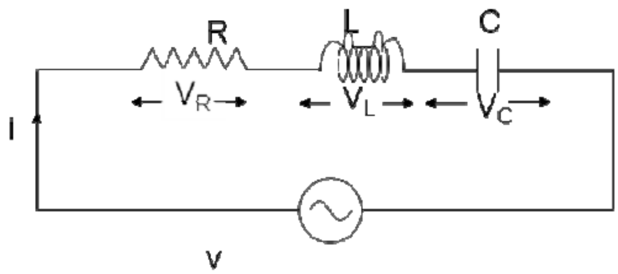




Phasor algebra for RC series circuit

$$V = V + j0 = V\angle 0, \bar{Z} = R - jX_C = Z\angle - \phi, \bar{I} = \frac{\bar{V}}{\bar{Z}} = \frac{V}{Z}\angle\phi$$

R-L-C Series circuit:



Consider an AC circuit with a resistance R, an inductance L and a capacitance C connected in series as shown in the figure. The alternating voltage v is given by

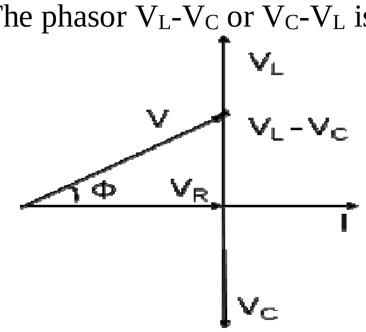
The current flowing in the circuit is i. The voltage across the resistor is V_R, the voltage across the inductor is V_L and that across the capacitor is V_C.

V_R=IR is in phase with I

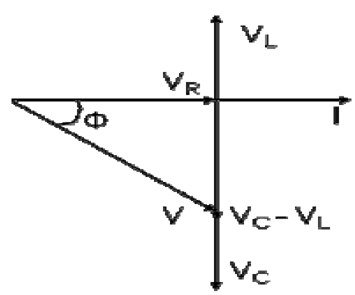
V_L=IX_L leads the current by 90 degrees

V_C=IX_C lags behind the current by 90 degrees

With the above information, the phasor diagram can be drawn as shown. The current I is taken as the reference phasor. The voltage V_R is in phase with I, the voltage V_L leads the current by 90^0 and the voltage V_C lags behind the current by 90^0. There are two cases that can occur V_L>V_C and V_L>V_C depending on the values of X_L and X_C. And hence there are two possible phasor diagrams. The phasor V_L-V_C or V_C-V_L is drawn and then the resultant voltage V is drawn.



V_L>V_C



V_L<V_C

From the phasor diagram we observe that when $V_L > V_C$, the voltage leads the current by an angle Φ or in other words the current lags behind the voltage by an angle Φ. When $V_L < V_C$,the voltage lags behind the current by an angle Φ or in other words the current leads the voltage by an angle Φ.From the phasor diagram, the expressions for the resultant voltage V and the angle Φ can be derived as follows.

$$V = \sqrt{V_R^2 + (V_L - V_C)^2} = \sqrt{(IR)^2 + (IX_L - IX_C)^2} = I\sqrt{(R)^2 + (X_L - X_C)^2} = IZ$$

Where impedance $Z = \sqrt{(R)^2 + (X_L - X_C)^2}$

Phase angle $\phi = tan^{-1}\left(\frac{X_L - X_C}{R}\right)$

From the expression for phase angle, we can derive the following three cases

Case (i): When $X_L > X_C$

The phase angle Φ is positive and the circuit is inductive. The circuit behaves like a series RL circuit.

Case (ii): When $X_L < X_C$

The phase angle Φ is negative and the circuit is capacitive. The circuit behaves like a series RC circuit.

Case (iii): When $X_L = X_C$

The phase angle Φ = 0 and the circuit is purely resistive. The circuit behaves like a pure resistive circuit. The voltage and the current can be represented by the following equations. The angle Φ is positive or negative depending on the circuit elements.

$$V = V_m sin\omega t, I = I_m \sin(\omega t \pm \phi)$$

Average power

$$P = VIcos\emptyset = (IZ).I.\frac{R}{Z} = I^2R$$

Hence the power in an RLC series circuit is consumed only in the resistance. The inductance and the capacitance do not consume any power.

Phasor algebra for RLC series circuit

$V = V + j0 = V\angle 0^0, \bar{Z} = R + j(X_L - X_C) = Z\angle\phi, \bar{I} = \frac{\bar{V}}{\bar{Z}} = \frac{V}{Z}\angle - \phi$

ELEMENTS OF ELECTRICAL CIRCUITS

UNIT - V (Network Theorems)

Objectives:

- To understand the significance of using various theorems in electrical circuits.
- To apply appropriate theorem to simplify the analysis of a network.

Syllabus:

Significance of network theorems, Superposition theorem, Thevenin's theorem, Norton's theorem, Maximum Power transfer theorem, Reciprocity theorem, Millman's theorem, Tellegen's theorem, Compensation theorem.

Outcomes:

On completion, the student will be able to:

- Apply principle of superposition to find combined response in an element.
- Obtain thevenin's and norton's equivalent for a given circuit.
- Determine the maximum power transferred to a load.

INTRODUCTION:

This chapter will introduce the important fundamental theorems of network analysis. Included are the Superposition, Reciprocity, Compensation, Thévenin's, Norton's, maximum power transfer, Millman's, and Tellegen's theorems. We will consider a number of areas of application for each. A thorough understanding of each theorem is important since it makes analysis easier.

5.1. Superposition theorem:

The superposition theorem can be used to find the solution to networks with two or more sources that are not in series or parallel. The most obvious advantage of this method is that it does not require the use of a mathematical technique such as determinants to find the required voltages or currents. Instead, each source is treated independently, and the algebraic sum is found to determine a particular unknown quantity of the network. The superposition theorem states the following:

The current through, or voltage across, an element in a linear bilateral network is equal to the algebraic sum of the currents or voltages produced independently by each source.

- ➢ The superposition theorem extends the use of Ohm's Law to circuits with multiple sources.
- ➢ In order to apply the superposition theorem to a network, certain conditions must be met:
  - All the components must be linear, meaning that the current is proportional to the applied voltage.
  - All the components must be bilateral, meaning that the current is the same amount for opposite polarities of the source voltage.
  - Passive components may be used.
  - Active components may not be used.

To consider the effects of each source independently requires that sources be removed and replaced without affecting the final result. To remove a voltage source when applying this theorem, the difference in potential between the terminals of the voltage source must be set to zero(short circuit); removing a current source requires that its terminals be opened (open circuit). Any internal resistance or conductance associated with the displaced sources is not eliminated but must still be considered .Figure 5.1 reviews the various substitutions required when removing an ideal source, and Figure 5.2 reviews the substitutions with practical sources that have an internal resistance.

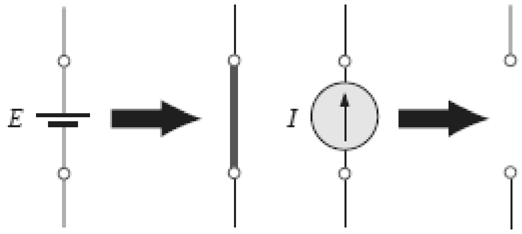


Figure:5.1 Removing the effects of ideal sources

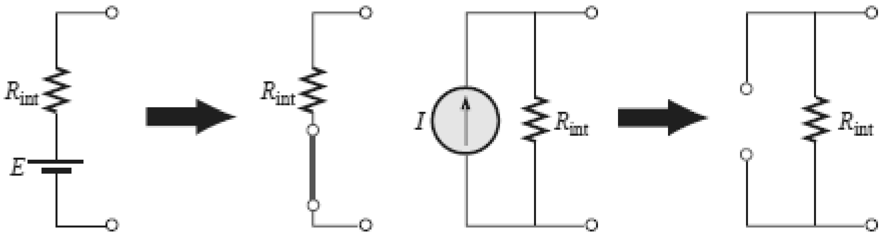
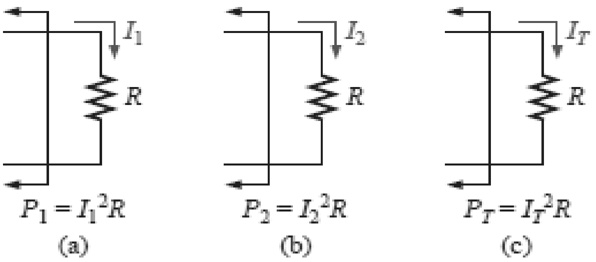


Figure :5.2 removing the effects of practical sources

The total current through any portion of the network is equal to the algebraic sum of the currents produced independently by each source. That is, for a two-source network, if the current produced by one source is in one direction, while that produced by the other is in the opposite

direction through the same resistor, *the resulting current is the difference of the two and has the direction of the larger.* If the individual currents are in the same direction, *the resulting current is the sum of two in the direction of either current.*

Figure 5.3:demonstration of the fact that superposition is not applicable to power effects



This rule holds true for the voltage across a portion of a network as determined by polarities, and it can be extended to networks with any number of sources. The superposition principle is not applicable to power effects since the power loss in a resistor varies as the square (nonlinear) of the current or voltage. For instance, the current through the resistor *R* of Fig. 5.3(a) is I_1 due to one source of a two-source network. The current through the same resistor due to the other source is I_2 as shown in Fig. 5.3(b). Applying the superposition theorem, the total current through the resistor due to both sources is I_T, as shown in Fig. 5.3(c)with

$$I_T = I_1+I_2$$

The power delivered to the resistor in Fig. 5.3(a) is

$$P_1 = I_1^2R$$

while the power delivered to the same resistor in Fig. 5.3(b) is

$$P_2 = I_2^2R$$

If we assume that the total power delivered in Fig. 5.3(c) can be obtained by simply adding the power delivered due to each source, we find that

$$P_T = P_1+ P_2 = I_1^2R + I_2^2R$$

$$P^2_T = I_1^2+ I_2^2$$

This final relationship between current levels is incorrect, however, as can be demonstrated by taking the total current determined by the superposition theorem and squaring it as follows:

$$I_T^2 = (I_1+I_2)^2 = I_1^2 + I_2^2+2I_1I_2$$

which is certainly different from the expression obtained from the addition of power levels.

In general, therefore, *the total power delivered to a resistive element must be determined using the total current through or the total voltage across the element and cannot be determined by a simple sum of the power levels established by each source.*

5.2. RECIPROCITY THEOREM

The reciprocity theorem is applicable only to single-source networks.

It is, therefore, not a theorem employed in the analysis of multisource networks described thus far. The theorem states the following:

The current I in any branch of a network, due to a single voltage source E anywhere else in the network, will equal the current through the branch in which the source was originally located if the source is placed in the branch in which the current I was originally measured.

In other words, the location of the voltage source and the resulting current may be interchanged without a change in current.

The theorem requires that the polarity of the voltage source have the same correspondence with the direction of the branch current in each position.

In the representative network of Fig. 5.4(a), the current *I* due to the voltage source *E* was determined. If the position of each is interchanged as shown in Fig. 5.4(b), the current *I* will be the same value as indicated. To demonstrate the validity of this statement and the theorem, consider the network of Fig. 5.5, in which values for the elements of Fig. 5.4(a) have been assigned. The total resistance is

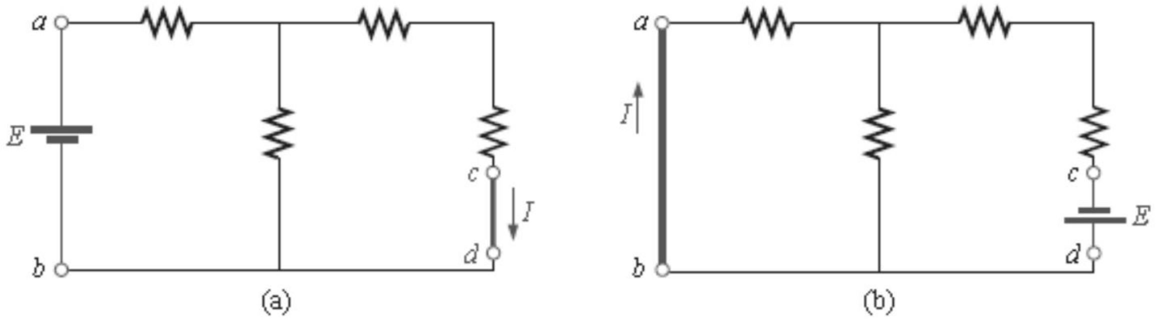


FIG-5.4

Demonstrating the impact of the reciprocity theorem

The uniqueness and power of such a theorem can best be demonstrated by considering a complex, single-source network such as the one shown in Fig. 5.5.

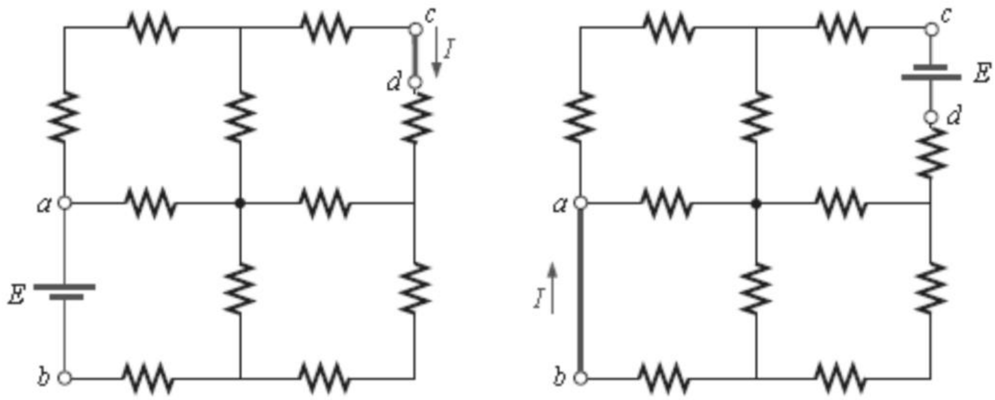


FIG-5.5 Demonstrating the power and uniqueness of the reciprocity theorem.

5.3 THEVENIN’S THEOREM

Thevenin’s theorem simplifies the process of solving for the unknown values of voltage and current in a network by reducing the network to an equivalent series circuit connected to any pair of network terminals.

Thévenin’s theorem states the following:

Any two-terminal, linear bilateral dc network can be replaced by an equivalent circuit consisting of a voltage source and a series impedance, as shown in Fig

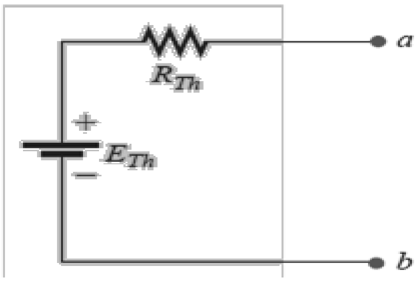


FIG 5.6 Thévenin equivalent circuit

In a given circuit except load impedance remaining circuit is to be replaced by a single voltage source in series with impedance.

The following sequence of steps will lead to the proper value of Z_{Th} and V_{Th}.

1. Remove that portion of the network across which the Thévenin equivalent circuit is to be found i.e, the load impedance Z_L is be temporarily removed from the network.

2. Mark the terminals of the remaining two-terminal network.

Z_{Th}:

3. Calculate Z_{Th} by first setting all sources to zero (voltage sources are replaced by short circuits, and current sources by open circuits) and then finding the resultant impedance between the two marked terminals. (If the internal impedance of the voltage and/or current sources is included in the original network, it must remain when the sources are set to zero).

V_{Th}:

4. Calculate V_{Th} by first returning all sources to their original position and finding the open-circuit voltage between the marked terminals.

Conclusion:

5. Draw the Thévenin equivalent circuit with the portion of the circuit previously removed replaced between the terminals of the equivalent circuit. This step is indicated by the placement of the impedance Z_L between the terminals of the Thévenin equivalent circuit.

5.4.NORTON’S THEOREM

The theorem states the following

Any two-terminal linear bilateral dc network can be replaced by an equivalent circuit consisting of a current source and a parallel impedance, as shown in Fig.

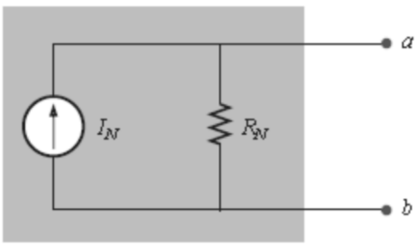


FIG-5.7 Norton equivalent circuit

The following sequence of steps will lead to the proper values of I_N and Z_N

1. Remove that portion of the network across which the Norton equivalent circuit is found.

2. Mark the terminals of the remaining two-terminal network.

Z_N:

3. Calculate Z_N by first setting all sources to zero (voltage sources are replaced with short circuits and current sources with open circuits) and then finding the resultant impedance between the two marked terminals. (If the internal impedance of the voltage and/or current sources is included in the original network, it must remain when the sources are set to zero.) Since $Z_N = Z_{Th}$, the procedure and value obtained using the approach described for Thévenin’s theorem will determine the proper value of Z_N.

I_N:

4. Calculate I_N by first returning all sources to their original position and then finding the short-circuit current between the marked terminals. It is the same current that would be measured by an ammeter placed between the marked terminals.

Conclusion:

5. Draw the Norton equivalent circuit with the portion of the circuit previously removed replaced between the terminals of the equivalent circuit.

* * *

Elements of Electrical Circuits

UNIT - VI (Network Theorems)

Objectives:

- To understand the significance of using various theorems in electrical circuits.
- To apply appropriate theorem to simplify the analysis of a network.

Syllabus:

Significance of network theorems, Maximum Power transfer theorem, Millman's theorem, and Tellegen's theorem.

Outcomes:

On completion, the student will be able to:

- Determine the maximum power transferred to a load.
- To Develop Milliman Equivalent Circuit.

INTRODUCTION:

This chapter will introduce the important fundamental theorems of network analysis. Included are the Superposition, Reciprocity, Compensation, Thévenin's, Norton's, maximum power transfer, Millman's, and Tellegen's theorems. We will consider a number of areas of application for each. A thorough understanding of each theorem is important since it makes analysis easier.

6.1. MAXIMUM POWER TRANSFER THEOREM

The maximum power transfer theorem states the following:

A load will receive maximum power from a linear bilateral dc network when its total resistive value is exactly equal to the Thévenin's resistance of the network as "seen" by the load.

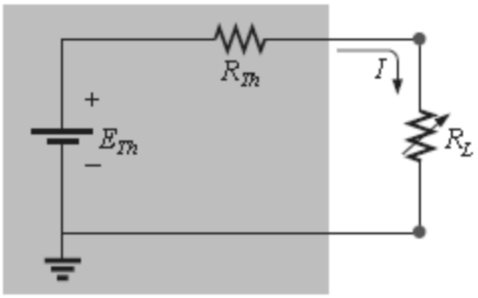


FIG-6.1

Defining the conditions for maximum power to a load using the Thévenin equivalent circuit.

For the network of Fig. .8, maximum power will be delivered to the load when

$$R_L = R_{Th}$$

From past discussions, we realize that a Thévenin equivalent circuit can be found across any element or group of elements in a linear bilateral dc network. Therefore, if we consider the case of the Thévenin equivalent circuit with respect to the maximum power transfer theorem, we are, in essence, considering the *total* effects of any network across a resistor *RL*, such as in Fig. 6.2.

For the Norton equivalent circuit of Fig. 6.2, maximum power will be delivered to the load when

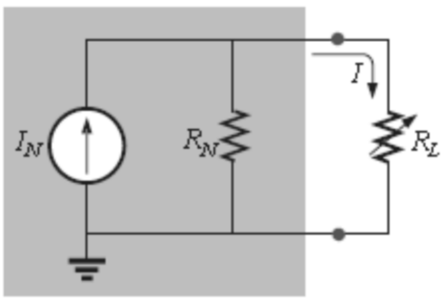


FIG-6.2

Defining the conditions for maximum power to a load using the Norton equivalent circuit

$R_L = R_N$

This result will be used to its fullest advantage in the analysis of transistor networks, where the most frequently applied transistor circuit model employs a current source rather than a voltage source.

For the network of Fig 6.1,

$$I = \frac{E_{Th}}{R_{Th} + R_L}$$

And

$$P_L = I^2 R_L = \left(\frac{E_{Th}}{R_{Th} + R_L}\right)^2 R_L$$

So that

$$P_L = \frac{E_{Th}^2 R_L}{(R_{Th} + R_L)^2}$$

$$I = \frac{E_{Th}}{R_{Th} + R_L} = \frac{E_{Th}}{2R_{Th}}$$

$$P_L = I^2 R_L = \left(\frac{E_{Th}}{2R_{Th}}\right)^2 R_{Th} = \frac{E_{Th}^2 R_{Th}}{4R_{Th}^2}$$

and

$$P_{Lmax} = \frac{E_{Th}^2}{4R_{Th}^2} (watts, \mathrm{W})$$

For the Norton circuit of Fig. 6.1

$$P_{Lmax} = \frac{I_N^2 R_N}{4} (\mathrm{W})$$

6.2 MILLMAN'S THEOREM

Through the application of Millman's theorem, any number of parallel voltage sources can be reduced to one. In Fig. 6.3, for example, the three voltage sources can be reduced to one. This would permit finding the current through or voltage across R_L without having to apply a method such as mesh analysis, nodal analysis, superposition, and so on. The theorem can best be described by applying it to the network of Fig. 6.3. Basically, three steps are included in its application.

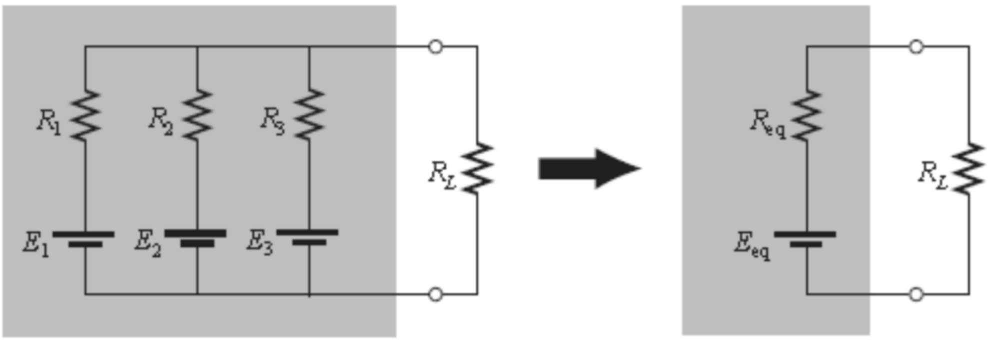


FIG-6.3

Demonstrating the effect of applying Millman's theorem

Step 1: Convert all voltage sources to current sources. This is performed in Fig. 6.4 for the network of Fig.6.4.

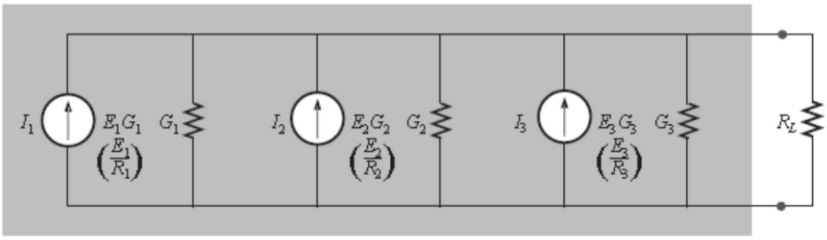


FIG-6.4

Converting all the sources of Fig. 6.3 to current sources

Step 2*:* Combine parallel current sources. The resulting network is shown in Fig. 6.5, where $I_T = I_1 + I_2 + I_3$ and $G_T = G_1 + G_2 + G_3$

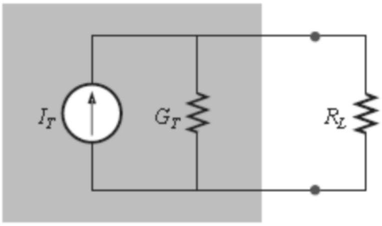


FIG-6.5

Reducing all the current sources of Fig.6.5 to a single current source

Step 3*:* Convert the resulting current source to a voltage source, and the desired single-source network is obtained.

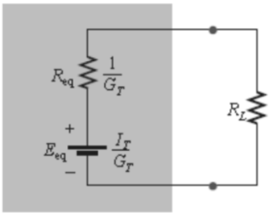


FIG-6.6 Converting the current source of Fig. 6.5 to a voltage source.

In general, Millman's theorem states that for any number of parallel voltage sources,

$$E_{eq} = \frac{I_T}{Y_T} = \frac{\pm I_1 \pm I_2 \pm I_3 \pm \cdots \pm I_N}{Y_1 + Y_2 + Y_3 + \cdots + Y_N}$$

The plus-and-minus signs appear in Equation to include those cases where the sources may not be supplying energy in the same direction.

The equivalent impedance is $Z_{eq} = \frac{1}{Y_T} = \frac{1}{Y_1 + Y_2 + Y_3 + \cdots + Y_N}$

In terms of the impedance values, $E_{eq} = \frac{\pm \frac{E_1}{Y_1} \pm \frac{E_2}{Y_2} \pm \frac{E_3}{Y_3} \pm \cdots \pm \frac{E_N}{Y_N}}{\frac{1}{Y_1} + \frac{1}{Y_2} + \frac{1}{Y_3} + \cdots + \frac{1}{Y_N}}$

and $$Z_{eq} = \frac{1}{\frac{1}{Y_1} + \frac{1}{Y_2} + \frac{1}{Y_3} + \cdots + \frac{1}{Y_N}}$$

The relatively few direct steps required may result in the student's applying each step rather than memorizing and employing.

6.3. Tellegen Theorem:

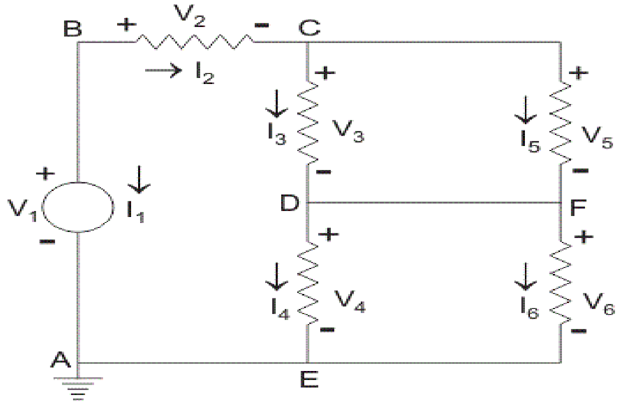
This theorem has been introduced in the year of 1952 by Dutch Electrical Engineer Bernard D.H. Tellegen. This is a very useful theorem in network analysis. According to Tellegen theorem, the summation of instantaneous powers for the n number of branches in an electrical network is zero.

Suppose n number of branches in an electrical network have i_1, i_2, i_3, ...i_n respective instantaneous currents through them. These currents satisfy Kirchhoff's current law. Again, suppose these branches have instantaneous voltages across them are v_1, v_2, v_3, ... v_n respectively. If these voltages across these elements satisfy Kirchhoff Voltage law then,

$$\sum_{k=1}^{n} v_k \cdot i_k = 0$$

v_k is the instantaneous voltage across the k^{th} branch and i_k is the instantaneous current flowing through this branch. Tellegen theorem is applicable mainly in general class of lumped networks that consist of linear, non-linear, active, passive, time variant and time variant elements. This theorem can easily be explained by the following example.

In the network shown, arbitrary reference directions have been selected for all of the branch currents, and the corresponding branch voltages have been indicated, with positive reference direction at the tail of the current arrow. For this network, we will assume a set of branch voltages satisfy the Kirchhoff voltage law and a set of branch current satisfy Kirchhoff current law at each node. We will then show that these arbitrary assumed voltages and currents satisfy the equation.



$$\sum_{k=1}^{n} v_k \cdot i_k = 0$$

And it is the condition of Tellegen theorem.

In the network shown in the figure, let v_1, v_2 and v_3 be 7, 2 and 3 volts respectively. Applying Kirchhoff voltage law around loop ABCDEA. We see that v_4 = 2 volt is required. Around loop CDFC, v_5 is required to be 3 volt and around loop DFED, v_6 is required to be 2. We next apply Kirchhoff current law successively to nodes B, C and D.

At node B let i_i = 5 A, then it is required that i_2 = − 5 A. At node C let i_3 = 3 A and then i_5 is required to be − 8. At node D assume i_4 to be 4 then i_6 is required to be − 9. Carrying out the operation of equation,

$$\sum_{k=1}^{n} v_k \cdot i_k = 0$$

we get, 7 X 5 + 2 X (− 5) + 3 X 3 + 2 X 4 + 3 X (− 8) + 2 X (− 9) = 0 Hence Tellegen theorem is verified.

* * *